Valuation and interest rate risk of mortgages in the Netherlands

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Preface

Writing a dissertation is a one-man sport that nevertheless requires the team-effort of many. Throughout my four year stay at the Finance Department of Maastricht University, I was surrounded by a team of people who all deserve a word of thanks for their warm encouragement and excellent input.

Jan-Willem Goslings was the first to spark my interest for financial research and to convince me that I was up to the challenge ahead. Many thanks go to him for getting me warmed up and started out on the right path. It was also thanks to him that *Assurantieconcern Stad Rotterdam anno 1720 N.V.* became interested in my research and I am very grateful for all of their financial support.

My largest debt by far is to Peter Schotman, whose excellent econometric skills, economic intuition and scholarly eye have provided me with the best constructive criticism possible. Many of the chapters of this dissertation have been shaped by his comments and advice. But I am most thankful to Peter for creating an atmosphere where I could be myself at all times, mistakes were allowed, Tibetan travel stories could be compared and the Smashing Pumpkins played in the background. Such things keep one whistling in the hallway.

Ronald Mahieu was another key supporter of this atmosphere. Despite the time pressures of finishing his own dissertation, Ronald found time to answer my numerous questions and play the role of assistant coach - not only academically, but also literally when I was training for the Rotterdam Marathon. I am pleased that Ronald and I were able to work on a research project together, the results of which you will find in Chapter 8. The data for that particular project were kindly provided by the Dutch Association of Real Estate Agents (NVM), with the generous help of Patrick Kerkhoffs, both of whom I am very grateful to.

Towards the end of my first year at the Finance Department, Dennis Bams joined our ranks and instantly became a good friend. Unlike myself, Dennis is a true econometrician and I appreciated his expertise on many occasions. But even more than this, I enjoyed our wide-ranging discussions at the student pub around the corner, frequently over a plate of satay.

I extend these thanks for a wonderful four years to my other colleagues at Maastricht University. Through all of you, indoor soccer and computer games acquired a new meaning! Christel Geus and Jaap Nijsen get a special thank-you for all of their hard work in assisting

me with the less glamorous side of research. The members of my academic committee - Piet Eichholtz, Angelien Kemna, Ton Vorst and Christian Wolff - provided comments and advice which were especially valuable during the last phase of research and writing.

Equally important to the support of my fellow academics has been the understanding and patience of those who are part of my non-university world. My family and friends have given me essential indirect support by keeping me happy, sane and well-fed.

Finally, I would like to thank Margaret Gold for her support, encouragement, and comments on various drafts of this dissertation. Most of all, I thank her for putting up with me while I was preoccupied with this project. I would like to assure her that I have no plans at this stage for future dissertations!

Arjan van Bussel
March 1998

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Chapter 1

Introduction

A mortgage loan is a long-term loan secured by registered goods, such as real estate, ships or aircraft. A mortgage contract authorizes the lender (the *mortgagee*) to sell the mortgaged property and foreclose the loan if the borrower (the *mortgagor*) fails to make the agreed-upon payments. That is, if the mortgagor defaults, the mortgagee has the right to sell the property in order to ensure that the debt is repaid. If the ensuing revenues are insufficient to cover the remaining mortgage debt, the mortgagee may join other creditors in laying claim to the debtor's estate.

Recent low mortgage rates have led to enormous growth in the number of loans issued to finance residential property purchases in the Netherlands. In 1996, the number of mortgages taken out by homeowners rose by 42 percent to 550 thousand (CBS, 1997). The amount outstanding on residential mortgages increased in 1996 by 10 percent to 379 billion guilders, such that the value of outstanding residential mortgages per capita in the Netherlands, with its 15.6 million inhabitants, is approximately 24,400 guilders.[1]

The increase in the total amount of mortgage debt is smaller than the guilder value of the newly issued residential mortgages because more than one third of the new contracts replaced existing ones. With replacement and prepayment playing such an important role, it is evident that much of the concern about mortgage valuation focuses on the propensity of mortgagors to prepay their loans prior to maturity. This prepayment option affects both the value and interest rate risk of the mortgage contract. Alongside the sharp increase in prepayment activity, the importance of an accurate mortgage valuation model is accentuated by the size and substantial growth of the market, the frequent introduction of new loan types, the substantial over-the-counter market and the growing interest in the public secondary mortgage market.

To value a mortgage contract properly it is essential to understand how mortgagors prepay in today's economic environment and how prepayment will fluctuate as economic

[1] At the end of 1996, the amount outstanding on American residential mortgages was approximately 3.4 trillion dollars. (Abrahams, 1997). On January 1^{st}, 1997, the US Census Bureau reported 266.5 million residents, such that the outstanding residential mortgage per capita in the US is currently over 12,750 dollars. The average dollar exchange rate in January 1997 was 1.80 guilders, which caused the American and Dutch per capita figures to closely resemble each other despite the differing homeownership rates.

conditions change. The prevailing approach on Wall Street is to isolate the determinants of past prepayments and to extrapolate into the future. Most of this empirical research is based on large mortgage pools constructed for securitization purposes.[2] This aggregation smooths out the individual loan characteristics that are behind the pool averages. In smoothing, much information is lost. One of the few exceptions is the research by Abrahams (1997), who focuses on individual incentives to prepay. For his study, Abrahams drew on historical prepayment data from more than 206 thousand individual loans. Such an extensive data set is not available for the Netherlands. For the empirical study described in Chapter 9, for example, we have prepayment details regarding a mere 333 Dutch mortgage contracts over a five and a half year period at our disposal.

The empirical relations found to hold in the US can not be directly translated to the Dutch mortgage market, as the differences are too large. The most noticeable is the contrast between the well-developed secondary mortgage market in the US and the modest size of this market in the Netherlands. The differences between the contract specifications in both countries are equally important. In the Netherlands, only 10 to 20 percent of the initial loan can be called in a calender year without penalty. Additional prepayments are settled at costs equal to the present value of the differences between future payments of the existing mortgage and a new contract. Annual prepayment restrictions are not common in the US, where the loan can be fully called without penalty.

Due to the annual prepayment limitations, curtailments (partial prepayments) have a much larger impact in the Netherlands than in the US. At the beginning of the life of an American mortgage pool, curtailments generally account for less than 1 percent of total prepayment (Patruno, 1994). The prepayment data studied in Chapter 9 shows that this is around 55 to 60 percent in the Netherlands.

Adjustable-rate mortgages whose contract rates are reset after a 5 to 10 year period are widespread in the Netherlands. After the fixed-rate period, the contract rate is freely reset to the prevailing mortgage rate. In the US, adjustable-rate mortgages contain cap and floor-restrictions to limit the degree by which the contract rate can fluctuate. Those restrictions partially offset the uncertainty for the borrower but increase the prepayment risk faced by the lender, *e.g.* whenever the embedded floor-restriction is binding, there is an incentive to replace the existing contract with a new loan.

Saving-to-repay mortgages are very popular in the Netherlands but virtually unknown in the US. These credit types do not require periodical repayments of the principal amount. Instead the borrower pays insurance and savings premiums on a regular basis. Upon mortgage maturity, the loan is repaid with the money saved in the savings account. Such an account commonly takes the form of a life insurance contract. Since no repayments take place during the term of these contracts, the outstanding balances remain untouched. Consequently, the interest costs to the mortgagors do not decline over the years. However,

[2] For example, Kang and Zenios (1992) used observations of several hundred thousand Mortgage-Backed Securities over an eight-year period to estimate their empirical prepayment model. Golub and Pohlman (1994) included over 28 million historical prepayment rates to calibrate the Wharton prepayment model.

interest paid on a mortgage is tax deductible in the Netherlands, and the return on the life insurance of a saving-to-repay mortgage is commonly not taxed. The combination of both tax facilities makes the prepayment of mortgages with life insurance unattractive.

Dutch mortgage contracts are often preceded by a quotation offer which embodies a minimum interest rate guarantee. During the months this offer is valid, the client can consider the loan conditions and is guaranteed that the lowest contract rate over that period will be honored. This minimum interest rate guarantee reduces the prepayment likelihood during the life of the mortgage.

The above analysis shows the larger prepayment risk faced by American mortgagees than that by Dutch lenders. However, the mortgage market in the Netherlands is rapidly becoming more dynamic and competitive. The increased competition has been prompting lenders to relax many of the prepayment restrictions and the risk of early receipt of principal is hereby becoming larger. As previously mentioned, this risk cannot be analyzed by looking at the refinancing experiences of the US or the Netherlands. Even if Dutch prepayment data was available, these observations would refer to past contract specifications. Any econometric model which fits this data would be outdated as soon as the prepayment restrictions are adjusted. As Lucas (1987) pointed out, it is dangerous to make inferences about the future based on past behavior for which the assumptions have now changed.

Hence, in order to analyze current Dutch mortgage contracts and assess the value and risk consequences of relaxing prepayment restrictions we must turn to a theoretical valuation model. An accurate mortgage pricing model consists of several components, of which the process to describe the interest rate dynamics is the most fundamental. The other necessary components are those which relate this process to the term structure of interest rates and mortgage rates, and those which address prepayment behavior.

The description of default behavior is of secondary importance in a theoretical model to value Dutch mortgages. Most studies which include the default possibility ignore the effect defaulting has on the credit rating of the individual. In the Netherlands a nationwide credit registry is kept which records current individual loans as well as those redeemed and defaulted within the last five years. Before a new mortgage is issued, the lender will consult this registry. Hence, the credit history of a potential mortgagor will influence the conditions against which he can borrow. Individuals who once defaulted face stricter conditions and acquiring a new mortgage becomes more difficult for them. Despite the fact that theoretical mortgage valuation studies do not include such extra costs they still conclude that default risk is of secondary importance to the lender. Including these extra costs will undoubtedly further reduce the relative importance of the default option. Consequently, default is omitted in the mortgage valuation models developed in the Chapters 5 and 7. Rather, we concentrate on the interest rate dynamics and the models which relate these dynamics to the term structure of interest rates and mortgage rates.

The choice of a particular interest rate model affects both the discount rates and opportunity costs for borrowers who consider refinancing. If the opportunity costs are lower than the coupon rate of the current mortgage, the homeowner has an incentive to replace the contract with a new one. The expected cash flow pattern therefore also depends on the interest rate model. Accordingly, the mortgage valuation results are sensitive to the interest rate model underlying the pricing algorithm. This thesis focuses on the impact alternative interest rate models have on the value and risk characteristics of a mortgage contract.

1.1 Outline

This thesis starts with an elaboration on the size and expansion of the Dutch mortgage market. Following this short overview, Chapter 2 briefly traces the most important characteristics of this market. The mortgage lenders and their market shares are discussed, as are the most popular loan types. The prepayment restrictions incorporated in Dutch mortgage contracts are introduced and the recently adjusted national mortgage guarantee program is summarized. Furthermore, Chapter 2 considers the taxation features of the different loan types and the recent developments on the secondary market for mortgages.

The literature summarized in Chapter 3 reviews the various factors relevant for mortgage pricing. The first part of this chapter focuses on theoretical pricing models, while the second part surveys the body of literature on empirical prepayment behavior. The overview reveals that no closed-form solutions exist to value fixed-income securities as complex as mortgages. For that we must resort to numerical methods as summarized in Chapter 4, where the strengths and weaknesses of the interest rate tree approach are considered in particular. With the help of simplified examples, this chapter introduces the principles of mortgage valuation on which the pricing algorithms developed in successive chapters are based.

The callable mortgage contracts studied in Chapter 4 are prepaid when the value of the mortgage, if left uncalled, exceeds the outstanding debt plus any transaction costs associated with refinancing the loan. This prepayment behavior is entirely determined by the dynamic process of the discount rate. There is no role for the mortgage rate at which the borrower can obtain a new loan. In Chapter 5 we compare this prepayment rule with one that depends explicitly on the mortgage rate at which the homeowner can refinance his loan on the market.

A second central topic of Chapter 5 is the impact alternative interest rate processes have on mortgage pricing. For this purpose, three empirical one-factor models for the short-term interest rate are specified. The dynamics of these single factor processes are described by discrete time, finite-state Markov chains. For mortgage pricing algorithms based on single factor interest rate models, the contract rate is a deterministic function of the short-term interest rate. For this functional relation, both exogenous and endogenous specifications are considered. The exogenous relations are based on historical observations in the Netherlands and the US. In the endogenous specification, the mortgage rate is determined such that the value of the contract equals the face value of the loan.

Although the short-term interest rate is the key variable in Chapter 5, other variables are frequently included as well. Commonly, these additional variables are related to long-term interest rates and mortgage rates.

Chapter 6 analyzes the empirical relation between the one-month interest rate, the long-term interest rate and the mortgage rate in the Netherlands. To study the dynamic interactions between these variables, Vector AutoRegressive techniques are used. We concentrate on the question whether the dynamics of the mortgage rate can be described by a one-factor interest rate model. The results indicate that a single factor does not correctly describe the interest rate term structure. Hence, to model the mortgage rate dynamics accurately more factors should be included.

In keeping with this, a multi-factor mortgage pricing model is developed in Chapter 7. The parameters resulting from the analyses in Chapter 6 are utilized to simulate short-term and long-term interest rates as well as mortgage rates. The resulting pseudo-histories of interest rates are used as the input of the valuation procedure.

One of the advantages of a valuation model based on a simulation procedure is that it can handle complicated contract specifications. In comparison with the contract analyzed in Chapter 5, two typical Dutch features are included in the contract studied in Chapter 7. The first one reflects the annual prepayment restriction. Secondly, we include the minimum interest rate guarantee frequently embodied in the quotation offer which precedes the contract. In order to compare the multi-factor approach with the single factor methods applied in Chapter 5, Chapter 7 also considers the standard contract without these additional features.

In Chapter 8 we make a side step and concentrate on the housing market. The developments in this property market are closely bound to the developments in the mortgage market. The vast majority of mortgage loans are secured by residential property, such that an in-depth analysis of the residential property market is appropriate here. In Chapter 8, we construct a repeat sales index to identify the general tendency of the Dutch housing market between May 1973 and December 1995. The index is based on data provided by the Dutch Association of Real Estate Agents (NVM).

The monthly return on this index is one of the independent variables in the regression analyses described in Chapter 9. In this chapter we study the prepayment behavior in the Netherlands in the late eighties and early nineties. For this we have historical prepayment data provided by *Assurantieconcern Stad Rotterdam verzekeringen anno 1720 N.V.* at our disposal. The observed prepayment behavior is compared with the prepayment activity as it would proceed from the prepayment rule applied in Chapters 5 and 7. The second part of Chapter 9 relates the observed prepayment rates to various variables suggested by economic theory as summarized in Chapter 3. These variables include housing prices, seasonality, aging, loan-to-value ratios, previous prepayments and prevailing mortgage rates.

Finally, Chapter 10 provides a summary and suggests directions for further research.

Chapter 2

The mortgage market in the Netherlands

2.1 Introduction

In recent years, mortgage rates have dropped to their lowest level in 25 years. At the end of 1996, rates were below 6%. These historically low rates have led to a substantial growth in the mortgage market in the Netherlands. Simultaneously, mortgage refinancing activities have intensified, interest in the public secondary mortgage market has increased and the variety of loan types has expanded. The purpose of this chapter is to introduce the basic aspects of the highly dynamic Dutch mortgage market.

To analyze the Dutch mortgage market the annual reports and quarterly reviews of the *Dutch Central Bank (DNB)* are extensively relied upon, as are the publications of the *Central Bureau of Statistics* and the mortgage guides of the homeowners association *Vereniging Eigen Huis.*

The Central Bureau of Statistics (CBS) publishes figures of newly issued, open registrations and redeemed mortgages. The mortgage statistics of the CBS are based on data recorded in mortgage deeds as kept by the land registry. The titles of the relevant CBS publications have changed over the years. From 1965 until 1975, the mortgage data were reported in *Hypotheek en hypotheekbanken.* Between 1976 and 1992, the figures were presented in *Statistiek der hypotheken.* And nowadays the mortgage figures are published in *Financiële maandstatistiek.*

Each year in March, the homeowners association Vereniging Eigen Huis publishes a Dutch-language mortgage guide (*hypothekengids*). The guide book discusses the various loan types, insurance and prepayment possibilities, tax treatment and government subsidies. Sections 2.4, 2.5 and 2.7 below borrow considerably from the 1994/1995 and 1997/1998 editions of this guide book.

2.2 Recent developments within the Dutch mortgage market

One of the key platforms of Dutch public policy is to ensure that sufficient adequate and affordable housing is provided. In order to achieve this, the authorities introduced two significant innovations in the late fifties. A tax deduction was created for the interest paid on mortgage loans, and a municipal guarantee program was established. The former stimulated homeownership at all levels of society and the latter eased the way for low-income earners to own their own homes.

These initiatives had a positive effect on the Dutch mortgage market which experienced considerable growth over the last decades. In 1950, 544,000 mortgages were outstanding. By 1965 this number was almost doubled, by 1980 it was quadrupled to 2.25 million contracts, and at the end of 1996, 4.34 million mortgages were outstanding, an 11.16 percent rise compared with the previous year. The rise in guilder value is equally remarkable. This is illustrated in Figure 2.1, where the amount of outstanding residential mortgages is compared with the Dutch Gross National Product and the market capitalization of the 130 largest companies quoted on the Amsterdam Stock Exchange. The amount of outstanding residential mortgages is the sum of the amounts outstanding at general, cooperative and savings banks plus the outstanding amounts of pension funds, insurance companies, mutual funds and social funds. These data are collected by the Dutch central bank on a quarterly basis.[1]

The Gross National Product time series in Figure 2.1 are annually reported by the IMF. The stock market capitalization time series are constructed by DataStream and are available on a monthly basis starting from January 1973. The same starting date was used as the basis point for the time series of the real amount of outstanding residential mortgages. For this, the nominal numbers were adjusted by using the seasonally adjusted IMF consumer price index.

At the end of 1996, a total of 379 billion guilders was outstanding on residential mortgages, an increase of 9.9 percent relative to the year before. The growth in the mortgage market becomes even more visible when the annual expansion is studied. For residential mortgages this is illustrated by the thick line in Figure 2.2. Here only mortgages outstanding at financial institutions under supervision of the Dutch central bank are included. This way it is possible to compare the estimated growth with the annual net call on the capital market concerning loans on houses and combinations house/industrial premises (the thin line). The net calls are based on residential mortgage data published in Table 7 of the annual reports of the Dutch Central Bank.

[1] Alongside financial institutes, other entities and individuals issue mortgages which are not reported to the central bank. To correct for this we assumed that the proportion of residential mortgages outstanding for these entities equals their 5-year average market share of newly issued mortgages, as collected by the CBS. For the years 1965-1996, their market share is illustrated in Figure 2.6 and discussed in Section 2.3. For the period 1957-1965 it is assumed that the market share of these non-financial institutes was constant at the same level as in 1965.

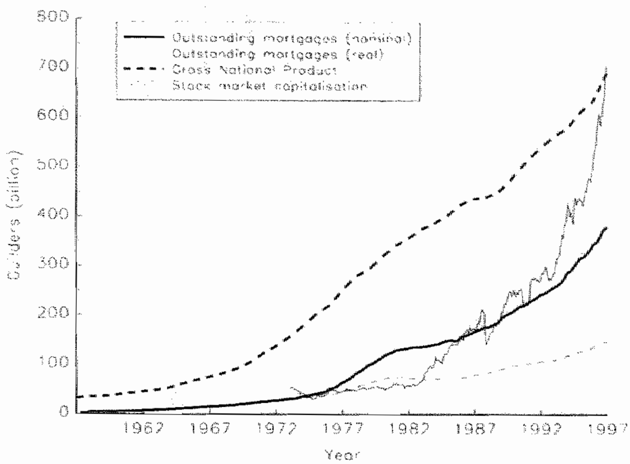

FIGURE 2.1: RESIDENTIAL MORTGAGES, STOCK MARKET CAPITALISATION AND GNP

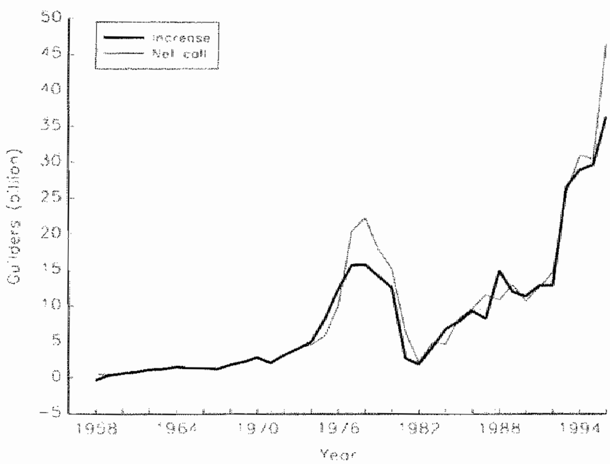

FIGURE 2.2: ANNUAL GROWTH OF THE DUTCH MORTGAGE MARKET

The time series of newly registered mortgages, plotted in Figure 2.3, also illustrate the recent expansion of the mortgage market in the Netherlands. Here the newly registered mortgages are categorized by nature of the mortgaged property. In this figure, the category '*houses & comb.*' contains both houses and combinations of residential and commercial space.[2]

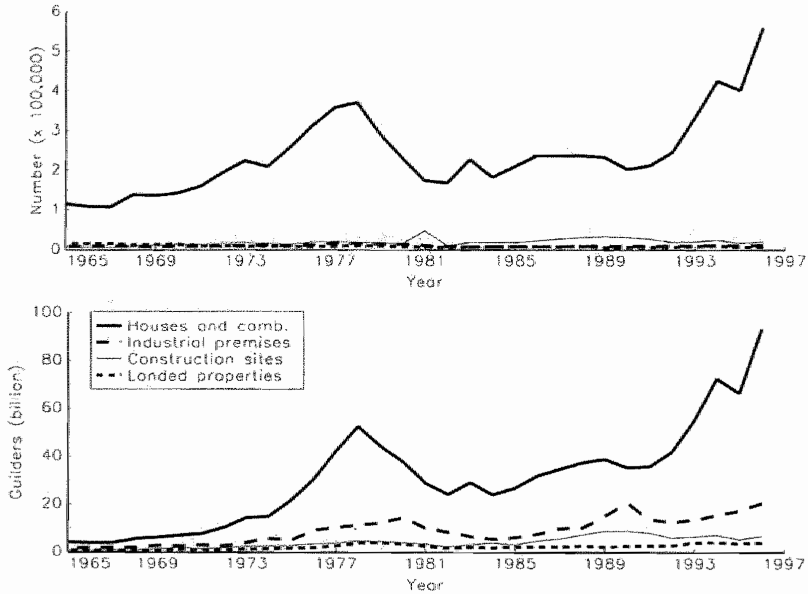

FIGURE 2.3: NEWLY REGISTERED MORTGAGES CATEGORIZED BY UNDERLYING PROPERTY

The upper diagram of Figure 2.3 shows the number of newly registered mortgages between 1965 and 1996. The overwhelming majority of mortgages on houses is clear. This also holds true for 1996. In that year a total of 600 thousand mortgages were issued, 92.87 percent concerned mortgages on houses and residential/commercial combinations, 1.83 percent referred to commercial industrial premises, and the construction sites and landed properties accounted for 3.25 and 1.07 percent, respectively. Not enough information is available to allocate the remaining 1 percent over the various categories. Looking at the principal amounts rather than the number of newly issued mortgages, the relative size of the house category decreases but remains by far the most important category. For the period 1965-1996, this is illustrated in the lower diagram of Figure 2.3.

[2] However, the other categories might also embody some homes, *e.g.* farmhouses and the porter's gate houses. Besides farmhouses, the category landed properties contains farmland, agricultural land, and rural properties such as barns.

Figures 2.1, 2.2 and 2.3 display a sharp rise in the size of the residential mortgage market in the mid seventies, a period which is known for the low -sometimes even negative- spread between the mortgage rate and inflation and the rapidly increasing housing prices. The relation between the residential mortgage market and housing prices becomes visible when the figures are compared with the price index for residential property in the Netherlands as constructed in Chapter 8. In the late seventies and early eighties the housing price bubble exploded. In real terms, housing prices dropped by more than 40 percent over a period of four years. In the same four-year-period the number of newly issued mortgages, as well as the total amount of granted mortgages, decreased by a similar amount. In 1982 this sharp decline in housing prices halted and the residential mortgage production stabilized at a level of around 200 thousand a year. In 1992 the number of newly issued mortgages started to increase again. In 1994, 424,700 residential mortgages were issued, and by 1996 this was already more than 550,000. This latter number represents a total amount of 93.132 billion guilders, such that the average principal was around 167 thousand guilders.

Housing prices in the early nineties also increased significantly, while at the same time the mortgage rate decreased from more than 9 percent to less than 6 percent by 1996. Consequently, a significant proportion of the newly issued mortgages reflects second mortgages and the replacement of existing mortgages with new mortgage loans. In 1993, 37,100 second mortgages were granted and 67,800 contracts were replaced with new mortgages.[3] These numbers measured 51,600 and 117,100 for 1994, and 49,500 and 90,600 for 1995, respectively. The low mortgage rate in 1996 had a significant influence on the numbers for that year. In 1996, 70 thousand mortgages were issued with a principal of less than 50 thousand guilders. Commonly, such small mortgages refer to second mortgages. In the same year a total of 190 thousand mortgages were replaced by a new loan with a lower contract rate. Due to these replacements, the total amount of outstanding residential mortgages increased much less than the figures of newly issued mortgage contracts would suggest. In 1996 the amount of newly issued residential mortgages was 42 percent higher than in 1995, while the total amount of outstanding mortgages was only 9.9 percent higher.

The time series of the amount of outstanding residential mortgages shown in Figure 2.4 concerns mortgages on houses and residential/commercial combinations. The total redemption is found by subtracting the annual increase in this economic variable from the newly issued contracts. This total redemption is plotted in the upper diagram of Figure 2.4 and is distinct from the total cancellations in the sense that the redemption includes total and partial repayments, while cancellations only refer to mortgages which cease to exist and as a consequence are removed from the public mortgage register.

The amount redeemed each year is in itself already interesting. Even more interesting is to look at it as a proportion of outstanding mortgages. This is illustrated in the lower diagram of Figure 2.4. The result is striking.

First of all, the redemption percentage is remarkably high. The maturity of most

[3] See Van De Beek and Vissers (1995), De Kruijk and Vissers (1996) and press release number PB97-39 of the CBS on February 6^{th}, 1997.

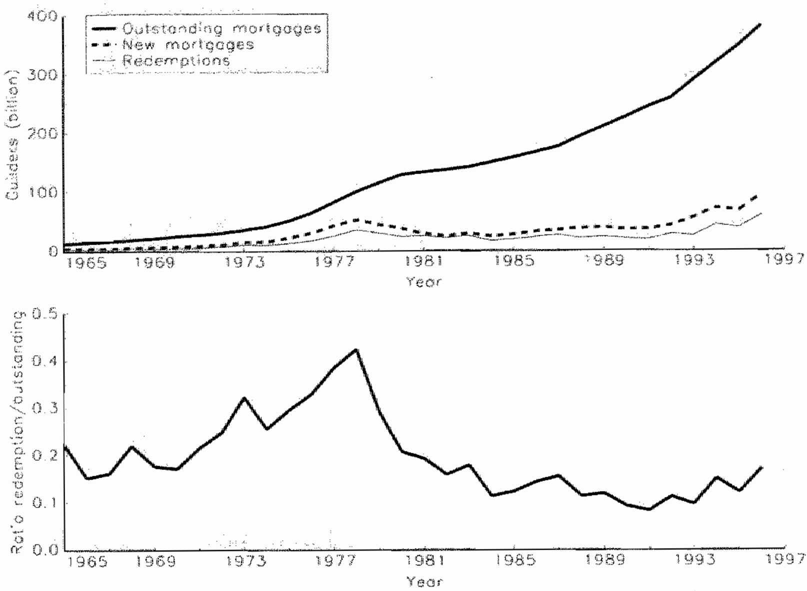

FIGURE 2.4: REDEMPTIONS, OUTSTANDING AND NEWLY ISSUED MORTGAGES

mortgages is 30 years, one would therefore expect that on average 3.33% of the outstanding mortgages would be redeemed each year. The lower diagram of Figure 2.4 shows that the realized percentage is much higher. In the seventies two redemption peaks can be recognized. These peaks, as well as the local minimums, can be explained with the help of the fluctuating mortgage rate. In Figure 2.5 the mortgage rate is plotted together with the seasonally adjusted inflation rate and the short and long-term interest rate. Up until 1970 the mortgage rate is only available on an annual basis, after which monthly average rates are at hand. These rates refer to annuity-mortgages. For inflation, the IMF consumer price index is used, here monthly data becomes available as of January 1972. From January 1970 to April 1980, the return on one-month loans to local governments is used as the short-term interest rate. From April 1980 on, the one-month Holland Interbank rate is used for this. Similar to the return on loans to local governments, this interbank rate refers to end-of-month observations. The long-term interest rate, as illustrated in the lower diagram of Figure 2.5, concerns government bonds with a remaining time to maturity of 5 to 8 years.[4]

The total redemption on residential mortgages as a percentage of the open registrations peak in 1973, just prior to a period of increasing mortgage rates. In 1974, when the mortgage rate rose 200 basis points within 7 months the relative redemption decreased.

[4] In Chapter 6 a more detailed description of these time series is given as well as an in-depth analysis of the interrelations between the mortgage rate and the short-term and long-term interest rates.

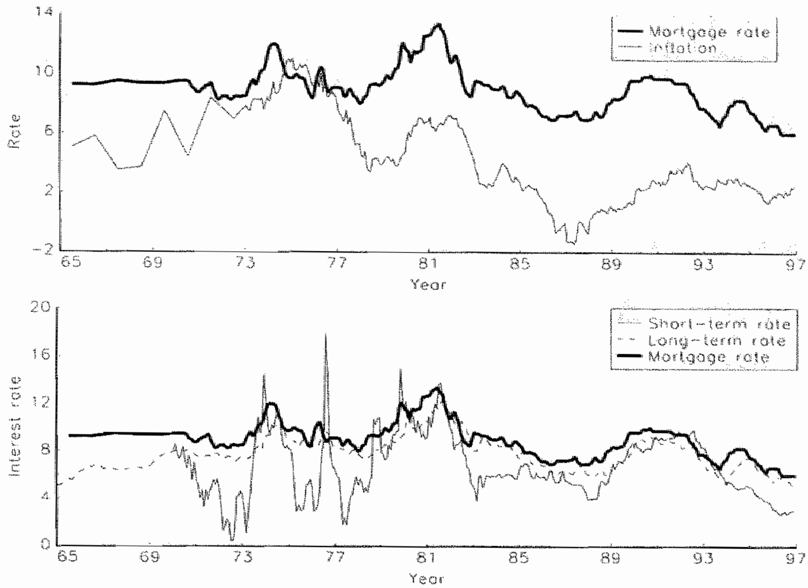

FIGURE 2.5: INFLATION AND INTEREST RATES

This suggests a negative correlation between the mortgage rate and the relative redemption. The observations of 1975 confirm this. In the last few months of 1974 and most of 1975 the mortgage rate decreased while the relative redemption increased. The correlation between the redemption rate and the mortgage rate is less clear in 1976. The redemption rate increased in respect to 1975, while the mortgage rate fluctuated like a yo-yo.

In the last quarter of 1976 the mortgage rate started to decline. In 1977 and for the most of 1978 this trend continued. In May and June 1978, the lowest mortgage rate in the seventies was reached, 8.02 percent. This coincided with the largest redemption rate of that decade. After that the mortgage rate started to increase. In November 1981 the highest mortgage rate in the considered period between 1965 and 1996 was reached, 13.45 percent. During this period of rising mortgage rates the redemption rate decreased. Once the mortgage rate began to decrease again, the redemption rate went up. Since 1983 the redemption rate has been relatively stable and low.

High redemption levels indicate a high activity of replacing one mortgage contract with another and can therefore be expected to occur at times when mortgage rates are low, for example in the mid-nineties. And as the upper diagram of Figure 2.4 illustrates, the redemption level increased during that period. Due to the low interest rates many new mortgages were issued in that same period. As a consequence the ratio redemption/outstanding mortgages, as plotted in the lower diagram of Figure 2.4, does not substantially increase.

2.3 The suppliers of mortgage financing

On the supply side of the mortgage market various parties can be distinguished. The largest group of mortgage originators are the general and cooperative banks, which issue mortgages as part of their total financial services package. Mortgage banks, on the other hand, specialize in granting long-term loans secured by registered goods such as real estate, ships and aircraft. Those activities used to be financed by issuing mortgage bonds.[5] Nowadays, most mortgage banks form a part of a large financial institute. This integration is often reflected in the attraction of funds. For example, over-the-counter loans from the mother institute have replaced the issuance of mortgage bonds to finance the lending activities.

Although cooperative, general and mortgage banks make up the largest proportion of entities participating in mortgage lending, other parties are active on this market as well. Savings banks and pension funds also account for a substantial market share. These financial institutes enter the mortgage market mainly for investment purposes. The main motivation for insurance companies to be active on this market is in order to stimulate the sale of connected life insurance contracts and to invest premiums in a safe and profitable way. The impact of building funds on the mortgage market is rather small and directly related to the houses built. Among the other entities with legal status which grant mortgages are mutual funds and employers. Finally, as illustrated in Figure 2.6, participation in the granting of mortgages is not limited to entities with legal status, however, the market share of entities without legal status (such as individuals) has become very small over the years.

Figure 2.6 shows the market shares of the major suppliers of mortgage financing in the Netherlands from 1965 to 1996. The figure is solely based on newly issued mortgages excluding open registrations. The large market share of the banks is especially distinguished.

2.4 Types of mortgage loans

The types of mortgage loans are divided into two broad categories: *fixed mortgages* and *equitable mortgages*. The most important characteristic of a fixed mortgage is that the borrower is obligated to make a predetermined series of payments, which might be adjusted when the contract rate alters.[6] Equitable mortgages, on the other hand, take the shape of a current account or checking account. From this account, the mortgagor can withdraw and redeem as he wishes, as long as an agreed-upon limit is not exceeded. This limit depends, among other factors, on the income of the mortgagor and the value of the property. The property is then usually appraised every five years, and the loan may generally not exceed 75% of that appraisal value. No redemption is required during the maturity of an equitable

[5] Mortgage bonds (*pandbrieven*) are fixed-income securities issued by mortgage banks which are very similar to standard bonds. See Rempt (1993) for an overview of the history of mortgage banks and mortgage bonds in the Netherlands.

[6] Fixed mortgages may not be confused with fixed-rate mortgages whose contract rate remains constant during the term of the loan.

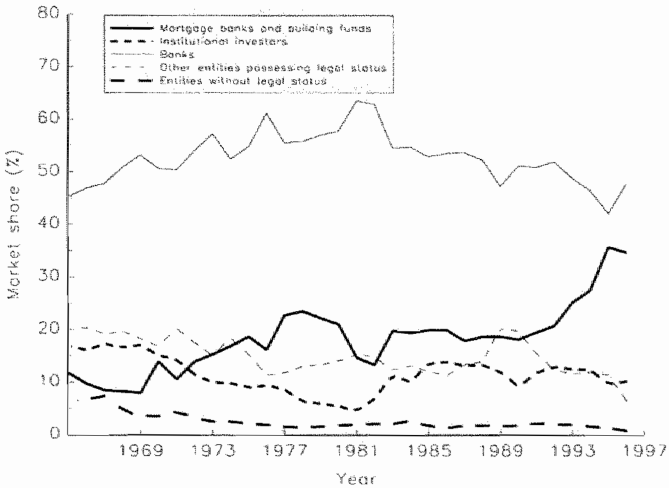

FIGURE 2.6: SUPPLIERS OF MORTGAGE FINANCING

mortgage, and interest is only paid on the amount actually withdrawn. This interest rate is variable and frequently reset.

In 1972 a new type of equitable mortgage, called a *bank mortgage*, was introduced. This vehicle also offers the borrower the opportunity to withdraw as desired, however, it differs from a standard equitable mortgage in the redemption of the loan. A redemption schedule is drawn up each time the borrower withdraws money from a bank mortgage. The most important characteristic of a bank mortgage is that the mortgaged property does not only secure the mortgage loan but also the other liabilities owed by the borrower to the bank. In Figure 2.7 bank mortgages are embodied in equitable mortgages. This figure illustrates the explosive growth of the Dutch mortgage market, distinguishing between fixed and equitable mortgages. The numbers are based on newly issued mortgages on real estate.

Mortgage loans can also be categorized according to their repayment schedules, distinguishing amongst three loan types.[7] The first type, the fully amortizing loan, is designed such that the amount of the outstanding mortgage balance is zero after the last scheduled periodical payment has been made. The second type, saving-to-repay mortgages, are coupled with a saving construction whereby the saved money will be used to repay the entire loan on the day the mortgage matures. The third category, non-repayment mortgages, is similar to saving-to-repay mortgages in that they do not require any repayments during the life of the mortgage. However, there is no savings account that corresponds with the loan. Redemption takes place in a lump sum financed by taking out a new loan, selling

[7] This categorization is based on the mortgage guide published by the homeowners association Vereniging Eigen Huis.

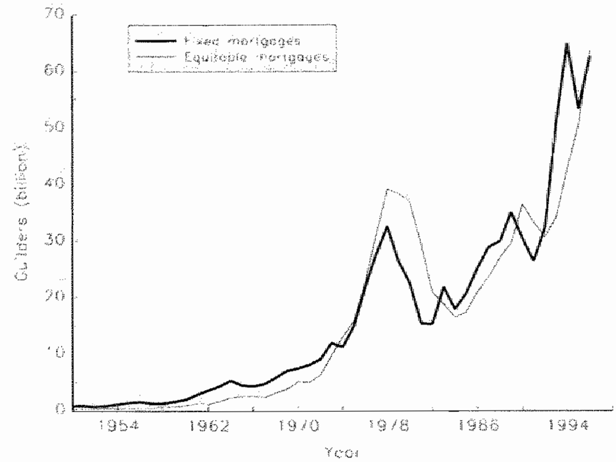

FIGURE 2.7: FIXED AND EQUITABLE MORTGAGES

the house, and so on.

In Figure 2.8 the fixed mortgages are subdivided amongst smaller mortgage categories.[8] As of January 1^{st}, 1994, 40 percent of the *outstanding* residential mortgage contracts were annuity mortgages, 15 percent were traditional mortgages with life insurance, 25 percent were savings mortgages, and the market share of linear redemption mortgages was 15 percent. The average remaining time to maturity of savings mortgages was 23.3 years, 18.9 years for annuity contracts, 17.6 years for traditional mortgages with life insurance and 16.2 years for linear redemption mortgages (CBS, 1996a).

Similar to Figure 2.7, Figure 2.8 is based on *newly issued* mortgages. However, while Figure 2.7 refers to real estate loans in general, independent of property and mortgagors, Figure 2.8 focuses on mortgages concerning loans on houses chargeable to individuals. The difference is remarkable. In 1992, a total of 30.65 billion guilders in equitable mortgages was granted on real estate of which only 9.06 billion guilders was to finance a house chargeable to individuals. This means that 70.43 percent of equitable mortgage financing was issued to bodies with legal status. This portion is smaller for fixed mortgages at 58.86 percent, with 41.14 percent being granted to individuals.

Figure 2.8 shows a steady growth in the popularity of mortgages combined with life insurance contracts. In 1992, the market share of this type of mortgage increased rapidly, probably due to a tax revision.

Even though many bank savings instruments were similar to contracts offered by life

[8] This distinction is only available for the years 1983-1992. Due to retrenchment policies, the CBS stopped collecting this information on such a disaggregate level. Currently, the CBS is preparing to resume the collection of detailed mortgage information on an annual basis.

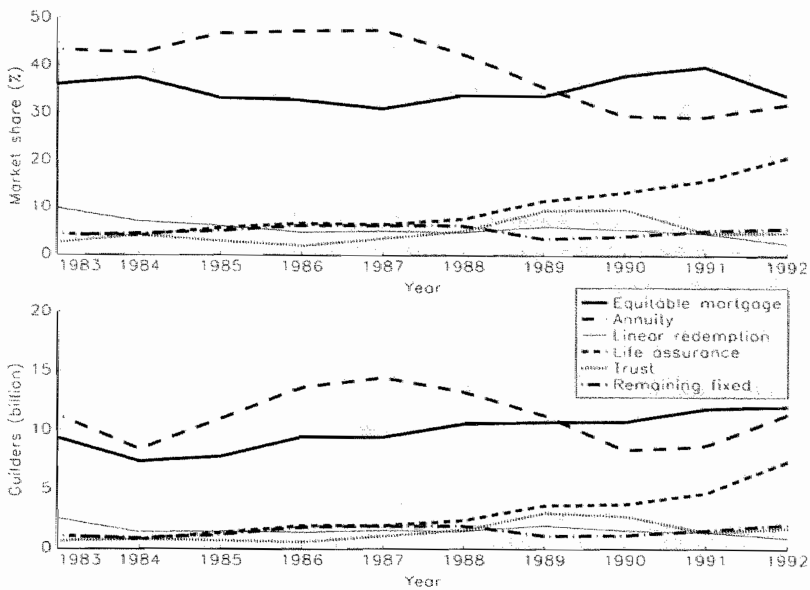

FIGURE 2.8: THE MARKET SHARE OF MORTGAGE TYPES

insurance companies, the income tax levied on payments was very different up until January 1^{st}, 1992. The tax revision *"Brede Herwaardering"* (Broad Revaluation) took effect on that date, the main purpose of which was to treat similar financial instruments in the same manner. As a consequence, the conditions for life insurance payments to be exempted from income tax narrowed down. Only payments upon death of the insured or upon mortgage loan repayment remained tax exempt. Under the new tax regime, life insurance contract payments resulting from other events are liable.

2.4.1 Fully amortizing mortgages

The basic idea behind fully amortizing mortgages is that the borrower periodically pays interest and repays a portion of the outstanding principal. At the maturity date of the contract, the outstanding balance is zero and the loan is fully repaid. The two most well-known examples of this category are the so-called *linear redemption mortgage* and *annuity mortgage*. The basic idea behind a mortgage with linear redemption is that the borrower repays a fixed amount per period, so that the debt gradually decreases. Due to the decreasing outstanding balance the interest payments decrease proportionally. Hence, the gross mortgage costs (interest plus repayments) decrease from period to period along a straight line. With an annuity mortgage, the periodical gross mortgage payments are constant over time. The interest payments decrease as principal repayments increase.

Besides linear redemption mortgages and annuity mortgages, there is a third loan type in this category of fully amortizing mortgages, called *fixed-wage mortgage*. This mortgage type is only issued by building funds to finance newly built homes. It is unique to the fixed-wage loan that it must be completely repaid before the borrower turns 66. However, the main characteristic of a fixed-wage loan is that the annually reset mortgage costs depend both on the interest rate and on the general wage index. A wage ratio is set for each mortgagor to fix the percentage of income used towards mortgage payment. This ratio is based on the general wage index rather than the individual income. As a result the mortgage costs rise when national aggregate wages rise, and vice versa. The relative number of fixed-wage mortgages is negligible in the Netherlands.

2.4.2 Saving-to-repay mortgages

Mortgages in this second category all share the characteristic of opening a savings account at the time the mortgage is issued. Under such mortgages no principal is repaid during the term of the contract. Instead the borrower makes payments on a regular basis to the lender which comprise interest on the mortgage loan, an insurance premium and a savings element. Upon mortgage maturity, the loan is repaid with the money saved in the savings account. Commonly, such an account takes the form of a life insurance contract, although other constructions are possible, *e.g.* participating in a mutual fund or starting an annuity insurance.

In the Netherlands, the interest paid on a mortgage is tax deductible. This makes mortgages with life insurance very attractive for high tax-bracket income levels in particular. Since no repayment takes place during the term of the contract, the outstanding mortgage balance remains unimpaired. As a consequence, the interest cost to the mortgagor does not decline over the years. These high interest costs are tax deductible for the entire term of the contract, while the returns on the life insurance are not taxed as long as some fiscal conditions are met. These conditions are discussed in Section 2.7.

Traditionally, the lump sum payment at the end of the maturity of a mortgage with life insurance depends on the return of an agreed-upon investment benchmark. In practise, returns on stocks, bonds or mutual funds are among the most common benchmarks used. Alongside these measurements, the profitability of the insurance company is frequently used as the benchmark. However, independent of the utilized benchmark, a *traditional mortgage with life insurance* does not guarantee that at the end of the term the sum of the accrued returns and constant premium payments will be large enough to repay the entire principal amount. Only a minimum return, commonly 4 percent, is guaranteed on the savings premiums. However, mortgage contracts with life insurance are often established with a payment schedule based on expected returns on the savings premiums of more than 4 percent. In the event of disappointing economic performance these expected returns may not be realized, and the resulting lump sum payment be too small to repay the loan. This scenario would leave the mortgagor with a debt at the end of the term. On the other hand, more favorable economic conditions would leave the mortgagor with more money after repayment than expected. The same uncertainty holds when the life insurance contract is prematurely closed or when the insured person dies.

This uncertainty is absent in the *mortgage with improved life insurance* often referred to as the *savings mortgage*. With a savings mortgage, the borrower is ensured that at the end of the term the saved money is enough to repay the loan. In contrast to a traditional mortgage with life insurance, the premiums paid on a savings mortgage are variable and the payment at the end of the term is known with certainty. Due to this fixed lump sum payment, the premium payment will decrease with rising interest rates, and vice versa. And because the interest costs of a savings mortgage are equal to the interest compensation received on the corresponding life insurance contract, it has a counter-intuitive effect, such that whenever the mortgage rate is low, the periodical savings premium has to be high to reach the same lump sum payment at the end of the term. The periodical mortgage burden thus can increase with decreasing mortgage rates. As a consequence, there is a positive correlation between the popularity of this mortgage type and the mortgage rate, *i.e.* demand for this product is low when the rate is low.

2.4.3 Non-repayment mortgages

Non-repayment mortgages are free of redemption during the maturity of the contract. Since only interest has to be paid, the mortgage burden per period is low. At the end of the term, however, the entire principal has to be repaid in full. This lump sum can be financed by taking out a new loan or by selling the house.

The restrictions on the relation between the size of the loan and the value of the underlying property are very rigid for non-repayment mortgages. In general, the lender requires that the mortgage credit does not exceed 75% of the forced-sale value of the house.

The most well-known non-repayment mortgage is the equitable mortgage. As discussed in Section 2.4, with an equitable mortgage the borrower does not receive the entire loan at once. This is in sharp contrast with standard non-repayment mortgages.

Another variant of non-repayment mortgages, is the *mortgage-for-life with life insurance*. No redemption need be paid on this mortgage during the life of the borrower. To ensure that the mortgage will be repaid, the mortgagee requires a life assurance to repay the loan when the borrower dies. The borrower need only to pay interest and a life insurance premium for as long as he or she lives.

2.4.4 Alternative mortgages

The mortgage types discussed in the above subsections are amongst the most popular in the Netherlands. Of course there are many more mortgage types, but most of these loans resemble the standard contract types discussed here. Alternative mortgages generally consist of a standard contract plus one or more extra options. These various options are attractive to mortgagors who are able to foresee or expect certain events to occur. For example, a social housing project homeowner may receive a subsidy from the government which has to be pre-financed, making a compound mortgage an attractive option. Similarly, if the mortgagor expects to be able to quickly repay a portion of the mortgage, or if a larger credit limit is foreseen to be needed, a compound mortgage might be the best solution. Most individual wishes can be combined with a standard mortgage contract to create a hybrid-contract, but the mortgage costs often increase substantially as a consequence.

2.4.5 Complementary perspectives

This section takes a step back from the focus on the analysis of interest rate risk and the valuation of mortgages from the lender's perspective. Instead, we switch over to the borrowers point of view in order to illustrate the different loan types available on the Dutch mortgage market. There is a variety of choice amongst the different loan types, each with its own tax-characteristics, rights and obligations, but the resulting cash flow patterns are very similar. The differences between the various mortgage types are therefore much less pronounced for the lender than for the borrower. Whether a lender grants a fully amortizing annuity-mortgage or a savings mortgage will not substantially influence the periodical cash flows received. In the first case it consists of repayments plus interest while in the second case it is composed of savings premiums and interest. In essence, the resulting cash flow pattern is the same. The same holds true for the outstanding balance. The remaining debt of a fully amortizing annuity-mortgage decreases with time. For a savings mortgage, the debt does not change during the maturity of the contract. However, savings premiums are collected during the life of such a mortgage contract and since the interest reimbursement on these premiums equals the mortgage rate, the outstanding balance of a savings mortgage is equal to the remaining debt of a comparable annuity mortgage at any moment in time.

From the lender's perspective the key difference between both popular mortgage types is the level of prepayment risk. A savings mortgage is often chosen because of its tax-characteristics. Prepaying such a mortgage can eliminate these tax advantages. The tax features of the alternative mortgage contracts are discussed in more detail in Section 2.7. Due to the differing tax status, the prepayment risk of a savings mortgage is smaller than that of an annuity-mortgage.

2.5 Modifications and penalties

Since October 1^{st}, 1996, practically all mortgage suppliers subscribe to the *credit code of practise*. This code is drawn up in cooperation with the government, consumers' associations and advisory bodies. The provision of information by mortgage suppliers has to meet the minimum requirements laid down in this credit code. Furthermore, this credit code prescribes minimum requirements regarding offer and contract conditions. Despite this standardization, the variety of rules and conditions embodied in the various mortgage contracts is vast. This section gives a short, general overview of modification conditions and penalties, especially focusing on the opportunity to prepay the mortgage or to make an extra premium deposit.[9]

If the prevailing mortgage rate is lower than the contract rate, a wealth-maximizing borrower will consider the possibility to prepay the existing mortgage and open a new one. This is generally not a cost-free decision. Besides the administration and transaction costs, a penalty is often imposed. These costs are often equal to the present value of the difference between the future monthly payments of a new contract and the existing mortgage. Both the applied discount rate and the periodical payments of the new contract are hereby based on the prevailing mortgage rate of a contract comparable to the existing one. Sometimes an additional fixed amount between 250 and 500 guilders is added to this penalty. The credit code prescribes that variable penalty costs may only be charged on the amount exceeding the penalty-free prepayment limit. It also requires that at least 10 percent of the initial principal may be prepaid within any full calendar year without penalties, but various lenders offer a larger percentage, such as 15 or 20 percent. Mortgages secured by the national mortgage guarantee, as is described in Section 2.6, have an extra prepayment feature. If a lender requires that the mortgagor annually repays part of the mortgage, then the mortgagor has the right to prepay an additional amount without penalty. This amount permitted must be at least as large as the obligated annual repayments.

Alongside (partially) prepaying a savings mortgage, the borrower might want to consider the possibility of making an extra savings deposit. This can have three effects. First, the borrower can choose to decrease the future premiums. For this it is important to know when the lender adjusts the savings and risk premiums. Whether this happens immediately after the extra premium is deposited or just once a year makes a large difference. Instead of lower future premiums, it is also possible to leave the premiums untouched and shorten the term of the contract. Finally, an extra deposit can lead to a higher lump sum payment at the end of the term. Each alternative has different effects on the taxation of the premiums or the final lump sum payment.

Besides the annual partial penalty-free prepayment opportunity, there are situations in which complete prepayment is free of costs: the demise of the mortgagor, bankruptcy, reception of a fire-insurance benefit, and the sale of the house. The only other instance that prepayment is free, is when the mortgage rate is reset at the beginning of the next fixed-rate period. Dutch mortgages usually have a maturity of thirty years with the interest rate fixed for a period of between one and ten years. At the end of each fixed-rate period the mortgage rate is reset to the prevailing market mortgage rate.[10] Usually there are no caps or floors restricting the interest rate adjustments at the reset date so that the new

[9] For a more complete discussion of this material please refer to the mortgage guide as annually published by the homeowners association Vereniging Eigen Huis.

[10] Some contracts add the option that the borrower can decide to switch at any point during the last year (sometimes even the last two years) before renewal to the prevailing mortgage rate and choose a new fixed rate period.

contract rate is in conformity with the market rate.

When a client moves, most contracts offer the option to arrange a new mortgage at the same contract rate as the mortgage on the existing house. However, the principal of the new mortgage may not be bigger than the unpaid balance of the existing contract. Otherwise the new contract rate will be a weighted average of the old contract rate and the actual mortgage rate. The option to use the old contract rate will of course only be exercised if the mortgage rate at that time is higher than the old contract rate.

2.6 National mortgage guarantee

In the Netherlands, local governments have guaranteed mortgages since 1956. This municipal guarantee was replaced by the national mortgage guarantee on January 1^{st}, 1995. The national mortgage guarantee is an assurance towards the mortgage lender. Whenever the mortgagor does not meet his financial obligations and the mortgaged home is sold for a price which is less than the remaining debt, the guarantee foundation will cover the difference. However, the guarantee is a suretyship-contract rather than an insurance in the sense that the individual is still responsible for any resulting loss. This means that the debt does not vaporize after the foundation has paid the difference. Instead of owing the mortgagee, the individual is now in debt to the foundation. As a result of the extra certainty, the default risk faced by the mortgagee is eliminated. This is reflected by a lower mortgage rate. The discount depends on the loan-to-value ratio.[11] The larger this ratio, the larger the discount. On average the corresponding interest rate is 20 basis points lower than without a guarantee. Sometimes the discount is even as high as 50 basis points.

To qualify for the mortgage guarantee program, the cost of the house may not exceed 315,000 guilders.[12] On at least half of the mortgage periodical redemptions must take place. The other 50 percent of the loan may be non-repayable during the life of the contract. The costs of participating in the program are equal to 0.36 percent of the principal plus 40 guilders administration costs. Ignoring tax, these costs are equal to 760 guilders for a secured mortgage of 200,000 guilders. If we assume that the discount on the mortgage rate is 20 basis points and that the mortgage term structure is flat with a mortgage rate equal to 6%, as it was in the beginning of 1997, then the present value of the savings is equal to 4,306 guilders and hence much larger than the initial costs.

During the 40 years that the municipal guarantee existed, it secured more than 1.5 million mortgages with a cumulative principal of 120 billion guilders.[13] In total, 19,000 forced-sales occurred during that period with a total loss of 900 million guilders. The average loss on a secured mortgage was 600 guilders.

There is not much actual data regarding defaults available in the Netherlands. The foundation running the national mortgage program only reports 100 defaults for 1996. It

[11] The loan-to-value ratio compares the amount of the loan with the value of the underlying property.

[12] The conditions described in this section hold as of January 1^{st}, 1997.

[13] See Bassant (1994).

is important to note that this foundation only becomes involved when the sale revenue of the house is not enough to cover the remaining debt. Due to the sharp rise in house prices and the relatively low mortgage rate in 1996, the number of such problem cases remained low.

Eichholtz (1994, 1995) found that regional economic stability and diversity can explain regional mortgage default risk. Eichholtz had Dutch mortgage default data from 1983 to 1991 at his disposal for this analysis. Figure 2.9 contains an overview of these data. The data solely refers to mortgages secured under the municipal mortgage guarantee program which was valid at that time.

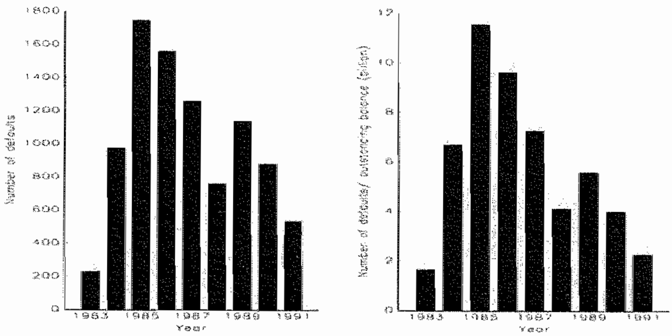

FIGURE 2.9: GUARANTEED MORTGAGE DEFAULTS IN THE NETHERLANDS
Source: Eichholtz (1994, 1995)

Between 1983 and 1991 a total of 9,096 guaranteed mortgages defaulted. The left diagram of Figure 2.9 illustrates that the number of guaranteed defaults increased rapidly between 1983 and 1985. In this period, the nominal price of houses hardly rose and the mortgage rate decreased, with the exception of the first half of 1983. As we discussed in Chapter 8, 1986 saw a slow but sure increase in house prices which rose more rapidly towards the end of the decade. The mortgage rate did not change much between 1986 and the end of 1988. For the same reason that the number of defaults in 1996 is low, the degree to which the municipal guarantee program needed to chip in declined over this period. In 1989 the mortgage rate started to increase, as did the number of defaults. Once this sharp rise in the mortgage rate halted, the number of defaults decreased once again.

Unfortunately, no data are available as to the total number of guaranteed mortgages outstanding during this period so that the relative importance of default risk can not directly be derived. As a proxy, we divide the number of defaults in a year by the average amount outstanding during that year. The resulting ratio is plotted in the right diagram of Figure 2.9 and shows the same fluctuations as the left histogram.

2.7 Taxation

The Netherlands is similar to many countries in allowing advisory, appraisal and administration costs to be tax deductible in the year the house is bought and the mortgage is issued.[14] Amongst other tax deductible items in the Netherlands are the cost of applying for the national mortgage guarantee, the cost of any required structural surveys, and the cost of the mortgage deed. Depending on the details of the situation, the handling fee, relocation expenses, and interest incurred during construction may also be deducted.

It is important to take note of a number of items which are tax deductible during the term of the mortgage contract:

- the interest paid on the mortgage,
- late payment penalty interest,
- prepayment and repayment costs, including penalty interest,
- the cost of mortgage registry annulment after the loan is completely repaid, and
- the cost of replacing the existing mortgage loan with a new one.

In contrast to non-mortgage loans, mortgage interest costs do not need to be used towards the tax liability on interest income. Saving-to-repay mortgages, with their simultaneous debt and savings characteristics make optimal use of this tax-deduction and exemption opportunity.

Section 2.4.2 introduced mortgages with life insurance, where the life insurance payment is not taxed as long as certain conditions are met. If the fiscal conditions are not met, the lump sum payment is taxed according to the graduated tax system in the Netherlands. To qualify for a tax-exemption, the life insurance payment may not exceed a predetermined amount of money. If the individual has paid premiums for 15 years, the largest of which does not exceed 10 times the size of the smallest, then a lump sum amount of 59,000 guilders is exempted from taxation. If, however, 20 years of premium payments were made, and the largest premium did not exceed 10 times the size of the smallest one, then the tax-free amount becomes 258,000 guilders. These restrictions hold for 1997, but the relevant limits are the ones which are valid when the lump sum payment occurs. The limits are adjusted for inflation such that a 30 year mortgage with life insurance issued today will have much higher limits in the future.

The tax-deductibility of interest paid on mortgages is frequently a topic ofdiscussion between politicians and tax-authorities.[15] The rationale behind the assessable rental value

[14] For an elaborated discussion (in Dutch) on tax matters related to buying, owning and financing a house see the mortgage guide published annually by the homeowners association Vereniging Eigen Huis.

[15] Recently, as described by Neuvel (1996), the Parliamentary Undersecretary of Finance prohibited the so-called continuing-construction. Under such a construction, a savings mortgage is not repaid when the accrued money on the life insurance contract is large enough. Instead the contract is continued either with

(*huurwaardeforfait*) is often drawn in this discussion. In the Netherlands, homeowners have to include a portion of the deemed rental value of their house in their annual income statements. Those who live in a fully-owned home must increase their taxable income by 1.25 percent of the sale-value of the house. Some authors advocate a higher percentage (such as De Kam (1996)), while others suggest abolishing both mechanisms simultaneously.

2.8 Securitization

Until recently, mortgage bonds offered investors the only possibility to be involved on the Dutch mortgage market. A mortgage bond is similar to a standard bank and debenture bond in the sense that it is a fixed-income security on which interest will be paid out. Mortgage bonds, issued solely by mortgage banks, are quoted on the Amsterdam Exchange and are therefore easily transferable. The return on Dutch mortgage bonds is independent of the cash flow pattern of the underlying mortgages issued by the bank. The mortgage bank will remain exposed to the risks involved in mortgage financing.

The Hooge Huys mortgage fund has been quoted on the Amsterdam Stock Exchange since October 18^{th}, 1993 and offers an alternative to direct investment in Dutch mortgages. This mutual fund invests at least 90 percent of its portfolio in residential mortgages. The remaining 10 percent can be invested in liquid assets and government bonds. More than half of the financed mortgages are guaranteed, while a maximum of 45 percent of the portfolio is invested in mortgages without guarantee. The payments following from the underlying mortgages are not directly passed-through to the participants in the fund, but annually drawn up in a profit and loss account. The Hooge Huys mortgage fund is a mutual fund which distributes the entire profit to its shareholders, such that the corporate tax is zero percent. Moreover, the shares enjoy infinite maturity in principal.

This is in sharp contrast with the mortgage pass-through securities that exist in the US. These securities are created when mortgages are pooled and sold to a legally independent entity, called a *special purpose vehicle*. This vehicle sells participations to the public, referred to as *Mortgage-Backed Securities* (MBSs). The risks inherent in mortgage investing are passed on to the investors through MBSs. The originator (or another institution which purchased this right) will continue collecting the interest and redemption payments. These payments minus the servicing, guarantee and other fees are passed on to the purchasers of such participations. This procedure is often referred to as *securitization*, which is not limited to mortgages. In the United States, credit card debt, taxi-permits and automobile-lease loan portfolios are also frequently securitized.

Mortgage-Backed Securities in Europe still assume modest proportions and are yet in

or without paying new premiums. In this way the interest on the mortgage remains tax-deductable at the same time as the extra savings are tax-free. However, the Parliamentary Undersecretary made an appeal to *fraus legis* to prohibit such constructions. Whenever the net-savings are larger than the net-costs, the continuing-construction is refered to as *fraus legis*, and the tax-deduction and exemption opportunities are cancelled.

their infancy.[16] The first European MBSs were issued in the United Kingdom in 1985. France and Spain followed in 1991 and 1993, respectively. MBSs have also been issued recently in Germany, Ireland and Belgium. In Austria and Luxembourg, MBSs are technically allowed but to the best of my knowledge have never been issued. The first MBSs were issued in the Netherlands on July, 24^{th}, 1996 by Fortis, a Belgian-Dutch financial conglomerate.[17] The size of this MBS issuance was 500 million guilders and the initial effective yield to maturity was around 20 basis points above a comparable government bond. The maturity runs from July 1996 to July 2006.

With securitization in mind, Fortis founded the special purpose vehicle F.I. Mortgage Securities B.V. (FIMS). This vehicle purchased a portfolio of residential mortgages from the VSB Bank N.V. and financed itself through the issuance of MBSs. Due to the securitization operation, the balance of the VSB Bank shortened by 500 million guilders which improved the asset-liability match. The ratio of equity-borrowed capital, and thus the solvability, also rose as a consequence of the transaction. For small, unhealthy financial institutions this can be an incentive to securitize assets. The major general and cooperative banks of the Netherlands have good solvability and no balance sheet problems. This holds to a lesser degree for insurance companies, who in the last two years have been able to sell more mortgages than they could finance. Hence these companies in particular may be expected to be interested in selling part of their mortgage portfolio on the liquid secondary mortgage market.

For an MBS market to be successful, a sufficiently large demand is needed. Whether this is the case in the Netherlands is not yet clear. The fact that all Fortis MBSs were sold within 15 minutes suggests that this is the case. Another indication is the fact that the idea behind Mortgage-Backed Securities has already been expanded to other assets. On December 18^{th} 1996, the Batavia Credit Card Corporation, an American financial institution, issued a 600 million guilder loan backed by credit card debt.

The existence of a good government guarantee program was a key factor in getting the MBS market started in the US. In the Netherlands the guarantee program is much more modest. Even though these residential mortgages are relatively standardized, this may be a handicap for the liquid secondary mortgage market. Due to the well-developed guarantee program, the debt risk is smaller for American MBSs than it is for Dutch ones. On the other hand, due to more flexible prepayment conditions, American mortgages have higher prepayment levels and therefore higher interest rate risks than their Dutch counterparts.

The success of the MBS market in the Netherlands will depend on the acceptance

[16] The first publicly traded mortgage pass-through security in the US was issued in 1970 on a 7.5 million dollar pool with a 7.00 percent coupon. The secondary mortgage market grew, after a modest start in the seventies, from 11 billion dollars in 1980 to 914 billion dollars in 1989. During the early nineties this amount rose even further, with nearly 1.7 trillion dollars outstanding by the end of 1996, such that almost one out of every two existing US mortgage loans is securitized. (Lowell (1995) and Abrahams (1997)) Only the US Treasury market is larger in volume.

[17] This discussion is based on circulars published prior to the issuance of the MBS by Fortis Investments (1996) and UBS Global Research (1996).

of these securities by institutional investors. A large over-the-counter market exists for mortgages among those investors. No exact data exists about the size of this over-the-counter market, but practitioners estimate that the annual turnover on this market is between 2.5 and 4.5 billion guilders. The future will show whether the investors prefer a liquid public secondary mortgage market over the current over-the-counter market.

2.9 Summary and conclusion

With the help of low interest rates, the mortgage market in the Netherlands has increased substantially in the recent past. At the end of 1996, a total of 379 billion guilders was outstanding on residential mortgages, up 9.9% from 1995. The number of mortgages refinanced more than doubled from 90.6 thousand in 1995 to 190 thousand in 1996. This means that about one third of newly issued mortgages in 1996 replaced older existing ones. This emphasizes the impact prepayment has on the mortgage market and the cash flow uncertainty faced by mortgage lenders.

The largest group of mortgage suppliers are the cooperative, general and savings banks, whose market share made up 47.8% of the Dutch in 1996. Looking at mortgage loan types, we see that fixed and equitable mortgages are equally popular. Within this group of fixed mortgages, the annuity contract is still the most predominant. However, due to their tax characteristics, the popularity of saving-to-repay mortgages has been increasing rapidly.

Historically, only between 10 and 20% of the initial Dutch mortgage loan can be called without penalty in a calendar year. Additional prepayments are commonly settled against costs equal to the present value of the difference between the future monthly payments of a new contract and the existing mortgage. This feature together with the taxation characteristics of saving-to-repay mortgages causes prepayment activity to be smaller in the Netherlands than in countries such as the United States.

In 1985, the first Mortgage-Backed Securities were issued in Europe. Recently, these instruments were introduced on the Dutch market. The first issuance had a value of 500 million guilders and was settled by Fortis on July 24^{th}, 1996.

Chapter 3

An overview of mortgage pricing

3.1 Introduction

This chapter presents the central concepts of mortgage pricing. Although the main part of the chapter is based on studies describing the US mortgage market, our primary interest is to raise general issues relevant in valuing a mortgage contract.

The majority of mortgage valuation studies tend to focus on a description of the term structure of interest rates as the key element in mortgage pricing. Many of them assume that all interest rates are solely driven by the instantaneous short-term interest rate. This implies that a deterministic relation exists between the short-term interest rate, the discount rates of the periodic cash flows and the mortgage rate. This assumption simplifies the description of the term structure of interest rates and its dynamics.

Although the short-term interest rate is the key variable in most studies, other variables are frequently included as well. These additional variables are commonly related to long-term interest rates, mortgage rates and house prices. Accordingly, this chapter starts in Section 3.2 with a discussion of alternative short-term interest rate processes and continues in Section 3.3 with a review of the effect additional variables have on mortgage pricing.

Sections 3.4 and 3.5 discuss various mortgage contracts. We begin in Section 3.4 with a default-free fixed-rate mortgage contract where the contract rate remains untouched during the entire maturity of the mortgage. Under the assumption of no-default, its value depends on the contract rate, time to maturity, other details of the contract and the prevailing discount rates which are determined by market interest rates. An investor in default-free fixed-rate mortgages faces two risks: the influence of fluctuating discount rates on the value of the contract, and the mortgagor's possible decision to prepay the contract when mortgage rates fall. As a consequence, the lender will incur a loss if interest rates increase, while the gain is limited when interest rates drop.

In actual fact, there is no such thing as a default-free mortgage, only mortgages which are insured. The effect which the default option has on the valuation of fixed-rate mortgages is discussed in Section 3.4.1. In Section 3.5 an adjustable-rate mortgage contract is considered. For such a mortgage the contract rate is periodically reset. The contract rate can usually not freely fluctuate due to restrictions by caps and floors. Consequently, the new contract rate not only depends on the prevailing mortgage rate at the reset date but also on rates prior to that date.

The US prepayment and default data show that many mortgagors fail to exercise the option to prepay when the prevailing mortgage rate is below the contract rate, whereas the option is frequently exercised even if the market rate is above the contract rate. This suboptimal termination behavior is discussed in Section 3.6. In Section 3.6.1 endogenous prepayment models which impose optimal termination behavior are extended with exogenous calls. Section 3.6.2 reviews empirical prepayment models. Section 3.7 concludes.

3.2 Interest rate processes

The effectiveness of a mortgage valuation model relies on the underlying interest rate process together with the theory on the term structure of interest rates. A distinction can be made between exogenous and endogenous term structure models.

Within the class of exogenous term structure models, the dynamics of the short-term interest rate are specified such that the resulting term structure exactly matches the term structure observed at the valuation date. Examples include the Ho and Lee model (1986), the Black, Derman and Toy model (BDT, 1990), and the model developed by Heath, Jarrow and Morton (1990, 1992). These models are partial equilibrium models in the sense that the initial term structure and the process generating shifts are determined exogenous to the model. In contrast with this, endogenous term structure models explain the movements in the term structure inside the model. The most frequently used endogenous interest rate models are one-factor models. The dynamics of the long-term interest rates for such single factor models are described by a deterministic function of the instantaneous short-term interest rate, r. Changes in this short-term interest rate are commonly described by the following continuous-time diffusion process:[1]

$$dr = \mu(r,t)dt + \sigma(r,t)dz, \tag{3.1}$$

where the functions μ and σ are the instantaneous drift rate and the volatility, respectively, and where dz is a Wiener process and t indicates the time period. The general diffusion process in applications is often specialized as a mean-reverting model where the interest rate, r, is pulled back to its unconditional mean, θ, with a speed-of-adjustment factor, κ:

$$dr = \kappa(\theta - r)dt + \sigma r^{\gamma} dz, \tag{3.2}$$

where $\kappa \geq 0$. Vasicek (1977) applied the Ornstein-Uhlenbeck process by letting $\gamma = 0$, accepting the possibility of r becoming negative. Courtadon (1982) assumed $\gamma = 1$. However, the most widely-applied specification of the mean-reverting process is the Cox,

[1] For an introduction into continuous-time finance see Shimko (1992). For an elaborated discussion see Merton (1994).

Ingersoll and Ross (CIR) model (1985a,b) , with $\gamma = \frac{1}{2}$. The CIR model is derived from a general equilibrium framework and has several attractive properties. For example, the variance is proportional to the level of the instantaneous risk-free rate. Low interest rates correspond with low volatilities. The disturbance term goes to zero as r goes to zero, and with $\kappa\theta \geq 0$ and $\sigma^2 > 0$ the drift term remains positive such that negative interest rates are excluded.

The effect alternative interest rate models have on the value of Mortgage-Backed Securities is studied by Chen and Yang (1995) and by Archer and Ling (1995). Chen and Yang (1995) compare valuation results using four widely-applied interest rate processes: (a) the Ornstein-Uhlenbeck process as proposed by Vasicek (1977), (b) the mean-reverting square root process developed by Cox, Ingersoll and Ross (1985b), (c) the log-normal process discretized by Rendleman and Bartter (1980), and (d) the Ho and Lee process (1986). The latter two processes are exogenous term structure models. Chen and Yang (1995) report that the actual shape of the term structure has a substantial impact on the accuracy of the pricing models. The Ho and Lee model turned out to provide the best empirical fit, while the log-normal process, without mean-reversion, yields the worst results. Furthermore, their results indicate that the volatility only plays a minor role in the pricing process and that possible negative interest rates do not harm the performance of the Ornstein-Uhlenbeck and the Ho and Lee model.

As Chen and Yang (1995), Archer and Ling (1995) reject the simple log-normal interest rate process for its poor fit with the term structure. Archer and Ling (1995) also included the CIR model and the no-arbitrage process of Black, Derman and Toy (BDT, 1990) in their study. The BDT model is also a log-normal interest rate model but is a more general model than the simple log-normal model in the sense that both the drift and volatility parameter are allowed to vary over time. This is because the current volatility curve is included in the log-normal BDT process which describes the distribution of future interest rates. Archer and Ling's comparative tests do not pin-point either the CIR or the BDT model as being more suitable for pricing mortgages and the prepayment option. The CIR model was found to yield higher option values than the BDT interest rate process. However, the results demonstrate that the effect the variance has on the option value is dominated by the mean-reversion characteristics of the underlying interest rate process.

3.3 Multi-factor interest rate models

Brennan and Schwartz (1978, 1979) raised the question whether the short-term interest rate is perfectly correlated with the long-term interest rate and the mortgage rate as in the single factor models. Instead they proposed to include a second variable in the valuation function of a default-free discount bond. Alongside the instantaneous rate, the long-term interest rate is embodied in their pricing model. Hereby they assumed that the two interest rates follow a joint Markov process. This assumption allows the derivation of a multi-dimensional partial differential equation to value a fixed-income security. A similar result is found in Section 3.4.1 where house prices are introduced as a second state variable. Another two-factor arbitrage model was developed by Schaefer and Schwartz (1984) who use the long-term interest rate together with the spread between the long and short-term interest rates.

Cox, Ingersoll and Ross (1985a,b) suggest a two-factor equilibrium model where an inflation process is added to the original one-factor interest rate model. Closely related to this approach, Longstaff and Schwartz (1992, 1993) developed a two-factor model where the instantaneous variance of changes in the short-term interest rate is added to the short-term interest rate.

The results by Litterman and Scheinkman (1991) suggests that three factors influence the term structure of interest rates. The three principal components which explain most of the variation in returns on all fixed-income securities are the level, steepness and curvature of the yield curve. In a study by Buser, Hendershott and Sanders (1990), the level of the short-term interest rate, as well as the slope of the term structure, stood out as a determinant of bond option values. Alongside these two variables, a general measure of interest rate uncertainty was included as a third fundamental factor. Although including more factors improves the modelling of interest rate dynamics, Buser, Hendershott and Sanders (1990) found that bond option models based on one-factor interest rate models produce price estimates similar to option values resulting from multi-factor models provided that comparable term structures and volatility levels are used. Hence, they conclude that with regard to bond option pricing

> "*...the number and nature of interest rate factors are largely irrelevant and so too is the reason that the term structure takes a particular shape.*"

This suggests that a one-factor interest rate model based on an exogenous term structure is adequate to price a mortgage and its prepayment option. This proposition is based on simulations, but due to the absence of analytical results Buser, Hendershott and Sanders temper their conclusion by not claiming that it holds for all possible specifications of multiple-factor models.

Canabarro (1995) also found that one-factor interest rate models work relatively well in pricing interest-rate derivatives such as bond options. Most mortgage valuation models are based on this result and use a single factor model to describe the underlying interest rate dynamics. In contrast, Brennan and Schwartz (1985), Schwartz and Torous (1989, 1992) and Boudoukh, Richardson, Stanton and Whitelaw (1997) argue that a second state variable is necessary for pricing mortgages and Mortgage-Backed Securities.

Following their earlier papers (1978, 1979) Brennan and Schwartz (1985) include the long-term interest rate as a second variable in their mortgage pricing model. They found that single factor interest rate models understate the value of the prepayment option compared to their two-state variable model.

Inspired by the tremendous growth on the secondary mortgage market in the US, Schwartz and Torous (1989, 1992) examined the valuation of mortgage derivatives. They employed a two-state factor model to describe the interest rate dynamics underlying the valuation procedure. Following Brennan and Schwartz (1985) the level of the short-term interest rate together with the yield on default-free long-term bonds are utilized by Schwartz and Torous (1989, 1992). The same two factors were used by McConnell and Singh (1993) to value Collateralized Mortgage Obligations. This is in contrast to Boudoukh *et al.* (1997), who assume that all information regarding the term structure of interest rates can be summarized by the 10 year yield and the spread between that yield and the 3-month T-bill rate. Using weekly prices for Government National Mortgage Association (GNMA) Mortgage-Backed Securities between 1987 and 1994, they found that a mortgage valuation model based on a single factor interest rate model is insufficient. Their results favor a valuation procedure based on both the short-term interest rate and the slope of the term structure.

3.4 Fixed-rate mortgages

A fixed-rate mortgage is a fixed-income security combined with a call option which gives the borrower the right to repurchase the mortgage at its book value at any time before the contract matures. Because of the callable features, interest rate fluctuations will not only influence the discount factor, but the projected cash flows as well. Declining interest rates will increase the probability that the mortgage contract will be prepaid. A rise in the market value of the contract is therefore limited, whereas higher interest rates result in a decreased market value, as with any fixed-income security.

A mortgage is a derivative asset in the sense that its value depends on interest rates and house prices. The value of the mortgage is just a package of possible pay-outs affected by the outcomes of these economic variables. Assuming no-default, the interest rate is the only relevant economic factor and the outcome of the package can also be attained by a portfolio of Treasury securities whose cash flows exactly mimic the expected flows of the mortgage. It follows from 'the law of one price' that the default-free mortgage has the same value as the portfolio or arbitrage opportunities would exist. This approach has been followed by Dunn and McConnell (1981a,b) and Buser and Hendershott (1983), among others.

3.4.1 Introducing default for fixed-rate mortgages

Early mortgage valuation models concentrate on the option to prepay the mortgage before it matures. Alongside prepaying the remaining principal, a borrower can terminate the mortgage contract ahead of schedule by exercising the option to default. Wealth-maximizing mortgagors will exercise this option whenever the market value of the house plus default costs fall below the book value of the mortgage. Cunningham and Hendershott

(1984), Epperson, Kau, Keenen and Muller (1985) and Titman and Torous (1989) ignore the prepayment option and focus on the default option. In more recent writings, authors have attempted to value both options simultaneously. Foster and Van Order (1985) were among the first to recognize the substitution effect between the prepayment and default option: by exercising one option the other is left worthless. Chen and Ling (1989), Leung and Sirmans (1990), Kau, Keenen, Muller and Epperson (1992) and Kau and Kim (1994) followed.

All papers on default have a second state variable in common, house price H, which is added to the single factor interest rate model. The fluctuations in house prices are hereby modelled as a continuous random walk:

$$\frac{dH}{H} = (\alpha - s)dt + \sigma_h dz. \tag{3.3}$$

Here α is the expected total return to owning a house. This return consists both of price appreciation and service flows from using the house, s. This imputed rental value of the house has to be subtracted from the total return to model the expected house appreciation $(\alpha - s)$. The disturbance term in Equation (3.3) is equal to the product of the house price volatility, σ_h, and a Wiener process denoted by dz.

The option pricing technique applied by Kau *et al.* (1992) and Kau and Kim (1994) among others, leads to a two-dimensional partial differential equation which describes the value of the mortgage contract. The default frequency implied by this differential equation does not correspond with the observed default numbers. Foster and Van Order (1985) empirically examined default behavior and found that mortgagors do not exercise the default option as strict as the economic theory prescribes. They attribute this discrepancy between theory and practice to large transaction costs. The findings of Vandell and Thibodeau (1985) are consistent with this. On the other hand, Kau *et al.* (1992), Kau *et al.* (1994) and Kau and Kim (1994) point out that these studies do not adequately account for the possibility to wait and exercise the default option at a later point in time. A rational mortgagor will not default as soon as the house price drops below the promised mortgage payments. By waiting to default, a mortgagor will profit from possible house price increases. This is, however, not free of cost. By postponing, one keeps the obligation to pay periodically the required mortgage payments.

Exercising the option or delaying default, both decisions are only driven by house price fluctuations. Even though the income, employment status and savings of an individual are among the factors that influence the default occurrence, neither Foster and Van Order (1985) nor Kau and Kim (1994) include personal characteristics of the mortgagor in the analysis. Moreover, negative equity may cause default regardless of the housing market or payment burden.

Alongside these personal features, the amount of the loan as compared to the market value of the property influences the default probability. This loan-to-value ratio is embodied in the studies of Vandell and Thibodeau (1985) and Kau *et al.* (1992). Their findings illustrate that high loan-to-value ratios increase the default probability. Furthermore, Kau *et al.* (1992) found that as long as the loan-to-value ratio and the house price volatility are not large, the

> "...*marginal contribution of default is small and a model without explicit default need not seriously misestimate a mortgage's value.*"

However, at the same time, they state that

> "...*for larger price volatilities the marginal value of default can be as large as the total value of prepayment.*"

They hereby applied volatility parameters in the same range as used by Titman and Torous (1989). The volatility is considered to be low when the annualized standard deviation of property returns is 0.10 or less, and high when 0.20 or more.

Both Leung and Sirmans (1990) and Kau and Kim (1994) conclude that the option to default is of secondary importance to prepayment. In the mutual interaction, prepayment turned out to have a far greater impact on default than default had on prepayment. When the loan-to-value ratio is low the default risk is small, even when the house price variance is high. The default risk only increases rapidly at high loan-to-value ratios (0.95 and higher).

The default of a fully-insured mortgage can result in principle in either a profit or a loss for the bank. However, if the market value exceeds the remaining unpaid principal the contract will be prepaid rather than defaulted. In the reverse case, the mortgagee will make a profit in the event the borrower defaults and the par value is paid by the insurer. Hence, a mortgage with full insurance has no default risk.

Many mortgages are partially-insured rather than fully-insured. However, the results of Kau *et al.* (1992) illustrate that there is hardly any difference in the value of a contract which is fully-insured and one whose coverage is limited to 20 or 25 percent of the loan. This is not only because it is unlikely that the value of the house will decrease that much, but also, and more importantly, because the mortgagor will have exercised the default option before such a significant fall in the house price can occur.

In conclusion most theoretical option-based studies assume that the mortgagor will default immediately after the value of the house falls below the mortgage value. Empirical studies tend to observe a delay in defaulting and generally attribute that to large transaction costs. Kau *et al.* (1992), Kau *et al.* (1994) and Kau and Kim (1994) argue that borrowers who do not want to sacrifice the right to default in the future, will make a rational choice to wait before defaulting.

Only some of the option-based models include mortgage features, like the loan-to-value ratio, in the analysis. Due to the difficulty of capturing divorce, death and unemployment in an econometric model, none of the studies include these important personal influences on default. In principle, transaction costs are easier to include in a valuation model despite the difficulty in identifying the true transaction costs of default. It is also very difficult to estimate the costs of the mortgagor who defaults and suffers a loss in his credit rating, making it much more difficult to get a new mortgage in the future. Moreover, foreclosure laws, not embodied in any of the aforementioned analyses, play an important role in practice. These laws, as do deficiency judgements, make defaulting much more expensive and decrease the value of the default option.[2]

3.5 Adjustable-rate mortgages

In September 1981 the rate on fixed-rate mortgages in the US reached a peak of 18% after an increase of 850 basis point during the preceding three years.[3] Mortgagees obtaining their funds through deposits of a short-term nature, as Savings and Loan banks, saw their mortgage portfolio devaluate rapidly while their liabilities increased. To prevent insolvency problems during such periods of highly fluctuating interest rates, financial institutions have to match the duration of the instruments on their balance sheet. This can be done by either lengthening the liabilities or shortening the assets. One instrument that satisfies this latter requirement is the adjustable-rate mortgage which was introduced in the US that same year.

An adjustable-rate mortgage (ARM) is a loan in which the contract rate is reset periodically in accordance with a suitable reference rate. The periodic adjustment of the contract rate shifts part of the interest rate risk from the lender to the borrower. Lenders frequently encourage individuals to choose ARMs over fixed-rate mortgages by offering a "*teaser*"-contract rate below the prevailing market rate.

The uncertainty for the borrower is partially offset by cap and floor-features embodied within the ARM contract. These features restrict the degree by which the contract rate can fluctuate between reset dates (periodic cap or floor) or during the entire life of the mortgage (lifetime cap or floor). Often both periodic and lifetime caps and floors are imposed in the ARM contract. In the US, ARM contracts are required to contain a lifetime cap, which is usually an out-of-the-money option bought by the borrower at the time the mortgage is issued. If a lifetime floor is embodied in the contract, the borrower sells an option to the lender at the same time. The value of the caps and floors depend on the benchmark index used to periodically reset the contract rate. At each adjustment date, the new contract rate is set by adding a margin to the prevailing level of the underlying index, taking the caps and floors restrictions into consideration along the way.

Hendershott and Shilling (1985) analyzed the margins required to earn the same return on ARM contracts as on fixed-income securities with a holding period of 7.5 years. These margins were found to depend on the slope of the term structure and the variance. A similar result was found by Buser, Hendershott and Sanders (1985) and by Huang and Xia

[2] Deficiency judgements give the lender the right to recover any deficiencies from the borrower's personal assets. In the US, only six states do not allow these judgements. See Fabozzi and Modigliani (1992) and Clauretie and Herzog (1989). In the Netherlands, a mortgagee, just as any creditor, is allowed to ask the bailiff to take possession of the borrower's assets if the latter fails to meet his obligations. However, the mortgagee has no priority when the assets are sold to pay out all creditors.

[3] See Cunningham and Capone (1990).

(1996) who indicate that mortgagors refinance their ARM contracts by moving into fixed-rate mortgages when the term structure flattens or becomes inverse.[4] An ARM contract will only be replaced by another ARM contract if the market rate is decreasing rapidly and the floor restrictions embodied in the old ARM contract prevent the contract rate from adjusting quickly enough.

The prepayment rate on ARM contracts is relatively stable and low compared to fixed-rate mortgages due to periodical adjustment of the contract rate to reflect the prevailing market rate. However, this is not true of the default rate. Cunningham and Capone (1990) illustrate that the shift of the interest rate risk to the borrowers results in higher default rates for ARMs than for fixed-rate mortgages and consequently advocate the inclusion of default probabilities in the valuation of ARMs.

The valuation of ARM contracts is complicated by the path-dependency of the contract rate. At an adjustment date, this rate depends not only on the prevailing mortgage rate but also on previous rates. When working backwards through the interest rate tree, these earlier rates are not available when necessary. Kau *et al.* (1990, 1993) overcame this impasse by introducing an auxiliary state variable.

The backward pricing approach starts at the maturity date of the contract, but, the contract rate and the unpaid principal of the ARM are determined earlier in time. To untangle this problem, Kau *et al.* (1990, 1993) observe that the value of the promised mortgage payments, the value of the prepayment option and thus the value of the mortgage are homogeneous of degree one in the unpaid balance. Therefore the problem can be solved for an arbitrary unpaid balance and rescaled as required.

3.5.1 Introducing the default factor into adjustable-rate mortgage valuation

Introducing default as a factor complicates the valuation of adjustable-rate mortgages because house prices become then also influential and have a direct effect on the option to default. Exercising this option leaves the prepayment option worthless. Consequently, the house price indirectly affects the prepayment option. Since both option values depend on the house price, their values are no longer homogeneous in the unpaid balance. However, each option value is homogeneous in the house price and unpaid balance together. This means that an equal change in the house price and the loan size is directly proportionate to the change in default and prepayment option values. Noting this makes it possible to calculate the value of the mortgage together with both embedded options.[5] However, as Kau *et al.* (1993) stress:

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[4] Some ARM contracts contain a conversion option which gives the borrower the right to convert their ARM into a fixed-rate mortgage. Non-convertible ARMs do not have this option but can be prepaid in favour of a fixed-rate mortgage through a normal refinance procedure, which of course is more expensive than exercising a conversion option.

[5] To determine the correct value numerical solution techniques are resorted to. The technique applied by Kau *et al.* (1993) is a two-state explicit finite difference method.

> *"It is important to observe that the technique used here to incorporate house price into the valuation technique is fundamentally different from the basic technique that suffices to value the promised payments. That basic technique, which extends to include prepayment on a default-free mortgage, in no way depends on the precise form of the interest rate process. However, the method used here to incorporate house prices works because the log-normal form assumed for the underlying asset H is a proportionate process, with the property that the rate of appreciation is independent of the size of the house."*

Empirical studies by Cunningham and Capone (1990) and VanderHoff (1996) confirm that ARMs are less frequently prepaid than fixed-rate mortgages. Exact figures are hard to come by and depend directly on the size of the rate caps and the length of the adjustment period. The default risk appears to be larger for ARMs than for fixed-rate mortgages. VanderHoff's results indicate that 60% of such defaults are sub-optimal from a financial point of view. His results also reveal that ARM holders are less apt to relocate. The teaser rate provides the opportunity to purchase a larger house which more closely matches their future housing demand and therefore decreases the probability of moving.

3.6 Prepayment behavior

Previous sections analyzed mortgage pricing models and results based on modern contingent claim techniques. Many of the studies described in Section 3.4 and 3.5 assume optimal prepayment and default behavior, in the sense that the mortgagor tries to minimize the market value of the loan. The resulting prepayment and default behavior is dependent only on the term structure of interest rates and house prices, thereby ignoring the individual characteristics of the borrower. These contingent claim techniques with endogenously determined termination are inappropriate for modelling observed prepayment and default numbers. Prepayment data on residential mortgages reveal that the prepayment option is frequently exercised when the prevailing mortgage rate is above the contract rate, while the mortgage is often not prepaid when it would be optimal to do so. Optimal call valuation models can not explain this behavior. The same holds true for the option to default.

Valuation models in which prepayments and defaults are exogenously specified override this empirical shortcoming. Exogenous prepayment valuation models can be divided into two categories. First, there are models which are based on endogenous models. These models take an optimal call and default valuation model as a starting point and add exogenous calls and defaults which are unrelated to the interest rate. On the other hand, there are strictly empirical prepayment models, which do not assume any optimal behavior. Instead, these models relate the observed prepayments to a set of explanatory variables.

3.6.1 Endogenous prepayment models extended to include exogenous calls

In optimal call valuation models, the prepayments are entirely endogenously determined. In such a model, the mortgage will only be called when it is financially optimal to do so. However, the prepayment option is often exercised when it is out-of-the-money. This means that the mortgage is prepaid even though the refinancing rate exceeds the contract rate on the existing mortgage. Dunn and McConnell (1981a,b) acknowledged this non-optimal behavior and incorporated it into a model in which prepayments are only interest rate driven with non-financial termination features. The model developed by Dunn and McConnell is based on the CIR term structure model. Dunn and McConnell augment this default-free model with a Poisson-driven process to explain the non-optimal prepayments. The mean of the Poisson process, which describes the probability that a non-financial termination will occur, is estimated from historical data. Dunn and McConnell emphatically assume that the so-called suboptimal prepayments are

> "*...uncorrelated with all relevant market factors and are, therefore, purely non-systematic.*"

Even though such exogenous terminations always increase the market value of the mortgage they are not necessarily irrational. Such behavior is often due to personal circumstances, such as job relocation or change in family size.

The Brennan and Schwartz valuation model (1985) adopts Dunn and McConnell's approach to include suboptimal prepayment behavior. Instead of using a one-factor model, as Dunn and McConnell do, Brennan and Schwartz use a two-factor model to value the mortgage and its prepayment option. They abstract from default, a possibility that is included in the two-factor model developed by Kau, Keenan, Muller and Epperson (1992). Unlike Brennan and Schwartz, they use house prices as the second state variable, as opposed to the long-term interest rate. The procedure applied by Kau *et al.* (1992) to value a mortgage under optimal call behavior alternates between the backward and forward-solving technique. Alongside this optimal termination behavior they also use a Poisson process to include suboptimal termination decisions. None of these models address transaction costs - a shortcoming that does not hold for the models developed by Giliberto and Ling (1992) and Archer and Ling (1993). As well as considering prepayments which occur when the option is out-of-the-money, Archer and Ling (1993) recognize that many mortgagors fail to exercise the prepayment option when this would be optimal. Their results indicate that transaction costs account for the observed lags in exercising in-the-money prepayment options. Stanton (1995) explicitly modelled heterogeneity in prepayment costs. He finds that the transaction costs implied in the observed termination data are significantly higher than the explicit monetary costs associated with refinancing.

3.6.2 Strictly empirical prepayment models

Strictly empirical models base the prepayment behavior only on a set of non-model-based explanatory variables. The prepayment decision is exogenously determined by models estimated from historical data. In such models, four main determinants are specified. The first and most important element that determines prepayment is the *refinancing incentive*. Homeowners tend to refinance the existing mortgage when the current mortgage rate is far enough under the contract rate. Different mortgagors face different refinance costs, this heterogeneity among households reappears in the prepayment data.

Not everybody will react immediately when faced with a prepayment opportunity. The most aware mortgagors will react and prepay their mortgage the first time a refinance incentive occurs. If the same refinance incentive occurs at a later stage, a smaller number of the remaining mortgagors will respond. The phenomenon of prepayment rates declining as mortgage pools age through interest cycles is known as *burnout*. The burnout effect is an aging effect, the older the pool of mortgages, the lower the prepayment rates.

Seasoning is also an aging factor but one with an opposite effect. When borrowers take out a new mortgage it is generally unlikely that the interest rate, family or employment circumstances change in the near future. The prepayment rates are therefore low at the beginning of a mortgage contract and increase gradually over time until they reach a stable or "seasoned" level. The American PSA (Public Securities Association) model is based on this idea. The PSA model assumes that the prepayment rates increase linearly during the first thirty months of the contract until they reach a 6% per year level, at which prepayment rates will remain constant.

Seasoning should not be confused with *seasonality*, which measures the correlation between prepayment rates and the month of the year. This cyclical behavior is highly linked with the cyclical behavior in house sales. Homeowners relocation tends to peak in the spring and summer and decline in the autumn and winter months.[6] Tax considerations and contract-specific constraints also play an important role in creating seasonality, particularly in relation to an increase in prepayment activity at the end of the year. Most Dutch mortgage contracts embody a maximum-prepayment-without-penalty clause for each year of the mortgage. If that maximum is not reached by the end of the year the option to prepay will loose its value, thus encouraging mortgagors to make full use of this option before it expires.

A macro-economic variable which is frequently mentioned as an explanatory variable is the house price. This variable is more useful in explaining prepayment behavior ex post than it is at enhancing long-term prepayment forecasts. If house prices were to be included in the prepayment forecast model, the variable itself would also have to be forecast. Hence, most empirical prepayment models concentrate on the four aforementioned factors: refinancing incentives, burnout, seasoning and seasonality.

There are several methods of applying historical prepayment numbers to project future

[6] See for example Richard and Roll (1989).

mortgage cash flows.[7] The traditional procedure to deal with prepayment uncertainties was to assume that the mortgage will be prepaid after a period which is equal to the average life of a portfolio of comparable mortgages. Instead of estimating the average life from historical data, the convention was to assume termination in 12 years. In practice, this pricing policy was not only applied on newly-issued mortgages, but also traders on the secondary mortgage market have even based their yields on this benchmark regardless the actual age of the underlying mortgages.

Curley and Guttentag (1977) recognized a life cycle pattern in the observed termination rates. They stressed the importance of using the entire distribution of termination probabilities rather than the average life rule-of-thumb.

Dunn and McConnell (1981a,b) compared both the average life rule-of-thumb and Curley and Guttentag's method with their own contingent claims model. In their valuation algorithm, Dunn and McConnell use a Poisson process to model non-interest-rate-driven prepayments while an optimal prepayment strategy is applied to model the finance driven prepayments. As previously mentioned, this is an endogenous prepayment model with exogenous calls as opposed to a strictly empirical prepayment model.

Rather than opposing optimal prepayment behavior, Green and Shoven (1986) posit that each period holds a possibility that the mortgage will be prepaid, depending on the age of the contract and on the exercise value of the call option. They construct a life-table for mortgages -similar to actuarial mortality tables- based on prepayment data collected regarding 4,000 mortgages in California from 1975 to 1982.

Green and Shoven employ a proportional hazard model to more easily divide the probability of prepayment into two multiplicative factors, the age of the contract, and the exercise value of the call option. This proportional hazard model reads:

$$\Gamma(a,t) = \Lambda(a) \cdot \Pi(x_1, x_2, \ldots, x_N), \tag{3.4}$$

where $\Gamma(a,t)$ is the turnover probability of a mortgage with age a at time t. The proportion of the mortgages that would be prepaid even under completely stationary homogeneous conditions is the "base-line hazard", $\Lambda(a)$. This base-line hazard measures the effect of mortgage age or seasoning on prepayment behavior. The exogenous factors, $x_1, x_2, \ldots, x_N$, determine whether the value of the function $\Pi(\cdot)$ is greater or less than one, indicating the increased or decreased prepayment likelihood as compared to the base-line hazard estimate. Equation (3.4) implies that the hazard function is memoryless, *i.e.* past figures (and also

[7] For an overview of the prepayment behavior of Federal National Mortgage Association (FNMA) Mortgage-Backed Securities see Becketti and Morris (1990). They observed that prepayment rates are related to the relative coupon and the age of the mortgage. Hereby, the relative coupon is defined as the difference between the prevailing mortgage rate and the contract rate on the existing mortgage. However, as Becketti and Morris note, modelling prepayments as a function of these variables is only useful if they account for a substantial part of the variability of the prepayments. Surprisingly, the single best factor for *explaining* prepayment rates, the relative coupon, has only very little ability to *predict* prepayments. The age and size of the mortgage pool are more succesful in accounting for the decision to prepay. However, the main contribution of Becketti and Morris' study is not in predicting prepayment rates, but in illustrating the different relations that might exist between various variables and observed prepayment rates.

expected future values) do not have any effect on present prepayments. This hazard model is proportional due to the assumption that exogenous factors have an equi-proportional impact at all mortgage ages. That is, if the exogenous factors make prepayment more likely at one age, they make prepayment more likely at any other mortgage age. Hence, factors that influence prepayments are time-homogeneous.

Similar to Green and Shoven, Schwartz and Torous (1989) do not oppose optimal prepayment behavior. In each state of the economy they assume a conditional probability that the mortgage will be prepaid. How large these probabilities are, is exogenously determined. They also use a proportional hazard model to estimate the relative importance of various variables to explain prepayment behavior. However, unlike Green and Shoven, Schwartz and Torous include several explanatory variables in their model: the effects of seasoning, interest costs savings from refinancing, lagged refinancing rates, and seasonality. For this they collect data for thirty-year Single-Family pools over the period January 1978 to November 1987. Since the sampled mortgages are guaranteed by the Government National Mortgage Association (GNMA), default can not be determined separately from prepayment.

Given the four explanatory variables and the GNMA prepayment data, Schwartz and Torous (1989) estimate the prepayment function. They find that the probability of prepayment increases significantly when the refinance rate is under the mortgage's contract rate, and that the seasonality parameter does not differ significantly from zero.

Schwartz and Torous integrate the estimated prepayment function into a Mortgage-Backed Securities valuation model, which is based on a two-factor model as in Brennan and Schwartz (1985). In their valuation model, Schwartz and Torous do not impose an optimal value-minimizing condition. The prepayments are fully based on empirical relations, and the Brennan and Schwartz' two-factor model is solely used to model the processes of the refinance and discount rates.

In their 1992 paper, Schwartz and Torous extend the above analysis by introducing a default possibility. Both the prepayment and default option affect the value of the mortgage contract, even if the mortgage is fully-insured. In this case, default by the mortgagor obligates the insurer to pay the outstanding principal. Although the effect is comparable to prepaying, the economic conditions are completely different. Schwartz and Torous focus on the pricing of default insurance, describing default with a hazard function and prepayment with a proportional hazard model. Unfortunately, due to lack of default data on individual homes, Schwartz and Torous do not estimate the default function parameters. In the prepayment model, the base-line hazard function is assumed to be consistent with the standard PSA prepayment rule-of-thumb, while the other parameters are based on the empirical analysis of Green and Shoven (1986). In 1993 , Schwartz and Torous applied a Poisson regression to estimate both prepayment and default based on extensive Federal Home Loan Corporation (Freddie Mac) data which was now at their disposal. The results indicate that the prepayment decision depends on the refinance opportunities and that prepayment and default differs greatly amongst regions. The age of the mortgage was also found to play a significant role in explaining the observed prepayment and default behavior. Moreover, default is affected by the initial loan-to-value ratio as well as by the volatility of the housing index returns.

Rather than using Poisson processes and proportional hazard models to estimate the mortgagor's financing decision, Richard and Roll (1989) advocate an alternative approach. In their model the four previously mentioned economic factors (refinancing incentive, seasoning, seasonality and burnout) are combined in a multiplicative formula to determine conditional prepayment rates (CPR):

$$\text{CPR} = \text{refinancing incentive} \times \text{seasoning} \times \text{seasonality} \times \text{burnout}. \qquad (3.5)$$

Among the alternative features of this model, one stands out in particular: Richard and Roll measure the refinancing incentive as the ratio of the contract rate to the refinancing rate as opposed to looking at the difference between them. In order to prepay the loan, the mortgage is retired and the remaining principal is paid off, thus giving up any embedded option. If the prepayment costs are proportional to the outstanding principal, then the value of this option per outstanding dollar depends on the ratio $\frac{\text{Present Value of an Annuity-Mortgage}}{\text{Outstanding Principal}}$. Richard and Roll show that if an annuity-mortgage is considered, this ratio can be approximated by $\frac{\text{Contract rate}}{\text{Refinancing rate}}$ and that option theory implies that it is rational to prepay the mortgage if this ratio exceeds some critical value.

Usually mortgagors need time to react to interest rate fluctuations. A variable developed to model refinancing incentives should include this. To most accurately express the refinancing incentive, the Richard and Roll prepayment model therefore uses a weighted average of recent ratio values.

The results show a highly non-linear relationship between the prepayment rate and the refinancing ratio. Whenever the ratio is less than one, the prepayment activity is relatively small, but increases rapidly as it exceeds one.

The seasoning effect is also a function of this ratio. However, this time only the most recent ratio value is considered. The larger the ratio, the faster a mortgage pool appears to season. It takes an average of around five years for a mortgage pool whose ratio equals one to fully season. Once a pool is seasoned, Richard and Roll assume that aging has no further effect on prepayment rates.

The third factor, seasonality, is included because it is generally assumed that prepayment rates are higher in the summer months than in the winter. Richard and Roll, however, find that the peak in prepaying occurs in October and November, while the prepayment rates are the lowest during February and March.

Different households face different refinancing costs, therefore the critical value of the ratio that triggers prepayment differs among households. And even for one household the critical ratio value is not constant over time. The heterogeneity among mortgagors and the fact that the critical value is not time-homogeneous is captured by the burnout factor. This burnout factor is path-dependent.

Richard and Roll found that roughly 95% of the prepayment differences over time and across coupons can be attributed to the four explanatory variables. This result indicates that the Richard and Roll model has a good fit to the data available. However, Richard and Roll do not discuss the actual functional relationships between the separate factors and the ratio $\frac{\text{Contract rate}}{\text{Refinancing rate}}$. How the factors proceed from the historical data is not mentioned. Kang and Zenios (1992) devote a large part of their paper to overcome this by describing the techniques for filtering the four factors out of the historical observations.

Golub and Pohlman (1994) compare the multiplicative model with other Wall Street models. They find that the multiplicative model, which they call the Wharton prepayment model, yields credible prepayment rate projections and includes all the essential economic variables which are commonly embodied in practitioners' models. Golub and Pohlman advise Kang and Zenios to use the ratio $\frac{\text{Present value of the mortgage}}{\text{Outstanding principal}}$ instead of $\frac{\text{Contract rate}}{\text{Refinancing rate}}$. The latter ratio does not capture the joint impact of actual mortgage rates and the remaining time to maturity. In a postscript to their paper, Kang and Zenios acknowledged that this adjustment did indeed generate better results.

3.7 Conclusion

This chapter gives an overview of the central concepts of mortgage pricing. The first being the importance of the stochastic environment of the mortgage. The fundamental stochastic variable underlying all mortgage studies is the short-term interest rate even though various authors argue that additional state variables are necessary to price mortgages and Mortgage-Backed Securities accurately.

The prevailing approach to price fixed-rate mortgages is to mimic its cash flows. No-arbitrage principles require that these cash flows have the same present value as the mortgage contract. This approach works well when fixed-rate mortgages are analyzed. The procedure becomes more complex when adjustable-rate mortgages are considered. The cash flows proceeding from such contracts depend on the history of the interest rates. Kau, Keenan, Muller and Epperson (1990, 1993) developed a methodology to overcome this path-dependency problem.

Studies which incorporate the possibility of default conclude that the marginal contribution of this option is small as long as the loan-to-value ratio is low and the house price volatility is not large either.

Finally, this chapter summarizes the body of literature published on empirical mortgage prepayment behavior. Here the assumption of wealth-maximizing borrowers was relaxed. Rather than imposing optimal call and default decisions, observed termination behavior is analyzed in these studies. Early empirical models are based on basic rules-of-thumb, *e.g.* the average life model and the PSA benchmark. However, as mortgage contracts became more complicated over the years and the mortgage market increased in size, prepayment behavior became far too important to rely uncritically on these rules-of-thumb. In the more recent studies a distinction can be made between those applying an endogenous prepayment model which is extended with exogenous calls and those who develop a strictly empirical model. Commonly, these empirical prepayment models specify four main determinants of prepayment activities. The first one captures the *refinance incentive* and is related to prevailing mortgage rates. *Seasonality* measures the variety in prepayment rates over months. *Seasoning* and *burnout* are both aging factors. Seasoning refers to the beginning of a contract, when the prepayment rates are low and gradually increase, while burnout describes the decreasing prepayment activities as mortgages age through interest rate cycles.

Chapter 4

Principles of mortgage valuation

4.1 Introduction

The mortgage market has become a highly dynamic market. The undeniable signs are the substantial growth of the market, the increasing interest for the secondary market and the sharp increase in the variety of loan types. The complexity of these newly developed contracts necessitates the use of numerical methods in mortgage pricing. In this chapter the principles of mortgage valuation are explained.

Following a short elaboration in Section 4.2 on numerical procedures, this chapter focuses on the general idea of interest rate tree methods by working out some simplified examples. Various interest rate derivatives will be priced by using backward recursion, and methods alternating between backward and forward techniques. The fundamental variable underlying all these numerical solution techniques is the short-term interest rate. Section 4.3 thus introduces a short-term interest rate model which is borrowed from Black, Derman and Toy (1990). Non-amortizing mortgages and mortgage-annuities are valued (both callable and noncallable) in Section 4.4. Similarly, callable and noncallable adjustable-rate mortgages are priced in Section 4.5. The pricing technique introduced by Kau, Keenan, Muller and Epperson (1990, 1993) is explained in more detail here. Section 4.6 studies a typical Dutch mortgage contract which is not fully callable, but limited to a percentage of the initial loan in each calendar year. Due to the path-dependency of the cash flows, this loan type cannot be valued by backward induction only. The problems that arise when such a mortgage contract is valued in a more realistic setting are explained in great detail in Section 4.6 and summarized in Section 4.7.

4.2 Numerical solution techniques

A mortgage valuation model consists of several major ingredients: a description of the interest rate dynamics, a model of the term structure of interest rates, the relation between mortgage rates and the term structure, and a model describing the prepayment and default behavior. Integrating these elements within one mortgage valuation model requires simplifying assumptions for computational reasons. For this we must compromise and weigh one simplification against another. A more elaborated model for the term structure involves simple prepayment and default behavior rules. On the other hand, sophisticated prepayment rules require a less complicated process to describe the interest rate dynamics.

The majority of mortgage valuation models, *e.g.* Dunn and McConnell (1981a,b), utilize a one-factor model to describe the interest rate dynamics. In a one-factor model all interest rates are driven by the instantaneous short-term interest rate, r. Based on the assumptions that (1) the single state variable r summarizes all relevant information, (2) the capital market is competitive and perfect, (3) trading takes place continuously and that the cash flows are paid continuously, and (4) individuals are non-satiated and behave in a rational manner, Dunn and McConnell (1981a,b) show that the value of a default-free fixed-income security, V, can be described by the following Partial Differential Equation (PDE)[1]:

$$\frac{1}{2}\sigma^2(r,t)V_{rr} + \left[\mu(r,t) - \lambda(r)\sigma(r,t)\right]V_r + V_t - rV + C(t,r,r_h) = 0, \qquad (4.1)$$

where $\mu(r,t)$ is the drift of the process, $\sigma^2(r,t)$ is the variance, and $\lambda(r)$ is the market price of interest rate risk. Subscripts on V denote partial derivatives. The fundamental state variable in Equation (4.1) is the instantaneous risk-free rate r. The variable $C(t,r,r_h)$ represents the cash flows at time t which depend on the prevailing short-term interest rate r and the history of the short-term interest rate, r_h. This path-dependency is a very important factor when pricing a typical Dutch mortgage contract with only restricted penalty-free prepayment possibilities. In the Netherlands, generally only between 10 and 20 percent of the initial loan can be called in a calender year without penalty. Path-dependency also complicates the valuation of adjustable-rate mortgages with caps or floors. In this case, the current contract rate is not only determined by the interest rate on the adjustment date but also by the rates before that. Kau, Keenan, Muller and Epperson (1990, 1993) introduced an auxiliary state variable to overcome this problem. This is discussed in Section 4.5, where a simplified adjustable-rate mortgage is analyzed.

There are an infinite number of functions that satisfy Equation (4.1). A unique solution requires boundary conditions which capture the characteristics of the mortgage contract: The value of the mortgage at maturity must equal its face value or the remaining principal. The value at maturity for a non-amortizing mortgage equals the initial loan, while for a fully amortizing mortgage this value is equal to zero, $V(r,T) = 0$. Secondly, the value of the mortgage contract must go to zero as the interest rate approaches infinity, $V(\infty,t) = 0$. These two conditions hold for each fixed-income security. A third constraint has to be introduced for a callable security in order to specify the interest rate at which the mortgage is called. Wealth-maximizing borrowers will choose the call strategy that minimizes the value of the mortgage contract. This strategy is driven by the stochastic process of the risk-free instantaneous interest rate. Ignoring transaction costs, optimal calls will prevent

[1] This PDE is the basic pricing equation of all securities, see Duffie (1996) for a detailed discussion.

the market value of the contract from exceeding the remaining unpaid principal of the loan, $V(r,t) \leq U(r,t)$.

Given these boundary conditions, Equation (4.1) can be solved to value a default-free fixed-rate mortgage. However, in order to accomplish this, numerical solution techniques are mandatory.[2] Here, three numerical solution techniques are considered. The finite difference method converts Partial Differential Equation (4.1) into a set of difference equations which will be solved iteratively. Such a set of difference equations is a discrete approximation of a continuous-time diffusion process commonly used to describe the interest rate dynamics.

Simulation methods are a second group of numerical solution techniques applicable to mortgage pricing. Simulation procedures are well-suited to value derivative securities whose payoff depend on the history of the underlying variable or when there are several underlying variables. Rather than solving the Partial Differential Equation (4.1) directly, simulation procedures utilize expected value calculations. In such procedures interest rate paths are simulated. Subsequently, cash flow patterns are allocated to those paths and the mortgage contract is valued for each path. An approximation of the true value of the mortgage contract is found by calculating the average value of the contract over all simulated paths.

In contrast to finite difference methods, simulation procedures are forward solution methods. The problem with forward pricing methods is the dependency of the prepayment decision at a specific point in time on the value of the contract in that situation. Simulation procedures inevitably require the use of an exogenous prepayment decision rule which does not always lead to optimal prepayment behavior. This problem is often bypassed by the third numerical solution technique based on interest rate trees. The general idea behind interest rate trees is illustrated in Figure 4.1.

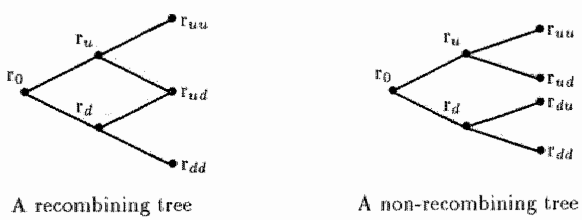


FIGURE 4.1: INTEREST RATE TREES

Figure 4.1 shows two alternative binomial interest rate trees. The interest rate at time $t + \Delta t$ in a binomial tree can retain one of two values depending on the outcome of a random draw, the probabilities of which are determined solely by the interest rate at time t. Two new branches arise from each of the possible interest rates at time $t + \Delta t$, and it only takes a few periods for the lattice to start looking like a tree (hence the name). The

[2] For an overview of numerical procedures see Duffie (1997) and Hull (1997).

interest rates from which new branches arise are called *nodes*. In a single factor model, the *state* of the economy can be summarized by these *nodes*, or *states* of the short-term interest rate.

The left diagram of Figure 4.1 shows a recombining interest rate tree. In such a tree, an interest rate path with an increasing rate in the first period and a decreasing rate in the second one has the same impact as a path where the interest rate first decreases and successively increases. This contrasts with non-recombining interest rate trees where the order of successive interest rate fluctuations is of great importance. This latter tree is suited for valuing contracts whose cash flows depend on the actual interest rate path.

Backward pricing methods are based on interest rate trees. Such methods evaluate the mortgage contract with its embedded options by starting at the end of the interest rate tree. At time T, when the contract matures, the value of the contract is known with certainty. This value is fed into earlier ones. The node value at time $T - \Delta t$ can be calculated as the expected value at time T discounted at rate r, which corresponds with the node under consideration for time $T - \Delta t$. This can be done for all nodes at time $T - \Delta t$. For each of the nodes it is necessary to check whether exercising the option is preferable to holding the contract. By working back through all nodes, the value of the mortgage contract at the moment of origination is found.

The major difficulty in using backward pricing methods is determining how to include the path-dependence of cash flows. Mortgage cash flows are path-dependent in the sense that prepayment not only depends on the current state, but also on the history of the interest rates. This is due to the simple fact that today's cash flow depends on the prepayment experience in the past.

4.3 Interest rate processes

Various mortgage loan types are valued in this chapter based on binomial trees. Before turning to numerical examples for illustration, we first need to introduce the interest rate model on which the valuations will be based. This chapter uses the Black, Derman and Toy (1990) model which describes a one-factor interest rate model and applies it to value Treasury bond options. The fundamental variable is the annualized one-period interest rate, which is assumed to be log-normally distributed. Given this assumption, Black, Derman and Toy (BDT) derive the future one-period interest rates which match today's term structure of interest rates and yield volatilities.

Table 4.1, which is borrowed from BDT, presents the figures of a fictitious term structure upon which all examples in this chapter are based. Under the risk-neutral measure the short-term interest rate one period from now can be determined with the help of this table: Today's value of a zero coupon bond with a maturity of 2 years is equal to 81.16 $(= \frac{100}{(1+0.11)^2})$. This must equal the expected price one period ahead, discounted back by using the one-period yield. BDT constrain interest rate fluctuations by use of a path-

TABLE 4.1: A FICTITIOUS TERM STRUCTURE

Maturity (years)	Yield (%)	Yield Volatility (%)
1	10	20
2	11	19
3	12	18
4	12.5	17
5	13	16

Source: Black, Derman and Toy (1990)

independent binomial lattice in which the short-term interest rate can only move up or down (with equal probability). Hence, the price one period in the future is either P_u or P_d, with P being price and the subscripts indicating whether the interest rate moved up or down, respectively. This is illustrated in Figure 4.2.[3]

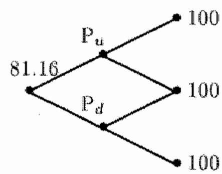


FIGURE 4.2: THE SHORT-TERM INTEREST RATE IN THE NEXT PERIOD

The present value of the expected bond price one period ahead must equal the present value of a two-year zero coupon bond, such that:

$$\frac{\frac{1}{2}P_u + \frac{1}{2}P_d}{1.10} = 81.16 \tag{4.2}$$

where $P_u = \frac{\frac{1}{2}100+\frac{1}{2}100}{1+r_u}$ and $P_d = \frac{\frac{1}{2}100+\frac{1}{2}100}{1+r_d}$. Clearly, r_u and r_d refer to the movement in the short-term interest rate. Assuming that in continuous time the short-term interest rates are log-normally distributed, the volatility of the two year yield in the applied binomial discreet approach must meet the following requirement:

$$\sigma_2 = \frac{1}{2}\ln\frac{r_u}{r_d} = 19\% \tag{4.3}$$

Equations (4.2) and (4.3) can be used to solve for the two unknown variables, $r_u = 14.32$ and $r_d = 9.79$. These results can be seen in Figure 4.3.

The next step is to find the three possible short-term interest rates for the next year. Once again two equations similar to Equations (4.2) and (4.3) can be derived. Unfortunately, this time there are three unknown parameters to make it appear that a unique

[3] Note that the interest rates reported in Table 4.1 refer to observed rates such that no risk-adjustment is required.

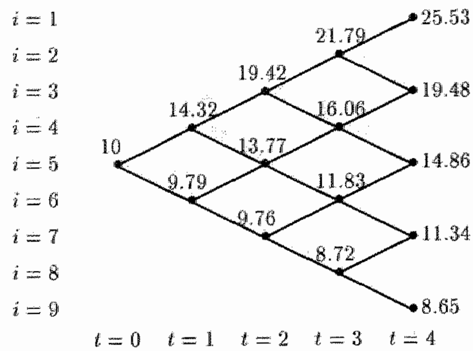


FIGURE 4.3: INTEREST RATE TREE

solution does not exist. However, the log-normality assumption regarding the short-term interest rate tells us that the volatility only depends on time. Whether the short-term interest rate went up or down in the first period does not matter. The volatility when the interest rate rises is $\frac{1}{2}\ln\frac{r_{uu}}{r_{ud}}$, while when the interest rate drops the volatility is $\frac{1}{2}\ln\frac{r_{du}}{r_{dd}}$. Since these volatilities have to be similar we know that $\frac{r_{uu}}{r_{ud}} = \frac{r_{du}}{r_{dd}}$. Because the interest rate tree is assumed to recombine we know that $r_{ud} = r_{du}$ and therefore $r_{ud}^2 = r_{uu}r_{dd}$. Hence the middle interest rate follows directly from the other two rates. This means that only two short-term interest rates have to match the two restrictions imposed by the three-year yield and volatility, the third one follows directly from this. Hence, a unique solution exists.

Applying this procedure to all nodes in different periods results in the interest rate tree plotted in Figure 4.3. This full tree of one-term interest rates corresponds with the term structure presented in Table 4.1.[4]

4.4 Fixed-rate mortgages

Fixed-rate annuity-mortgages are analyzed in this section, which builds upon the term structure of Table 4.1 and the dynamics of the short-term interest rate summarized in Figure 4.3. More precisely, the contract rate will be determined such that the value of the contract at origination equals the face value of the loan, which is assumed to be 100. In this setting, the contract is an interest rate derivative with a zero net present value. By doing this for comparable callable and noncallable securities, the option-adjusted spread is found.

Let us first consider a 4-year noncallable fixed-rate annuity-mortgage with an annual coupon of 11 percent. This example is graphically illustrated in the left diagram of Figure

[4] This section uses the Black, Derman, Toy approach to derive the interest rate tree. The same result has been reached by other authors through use of forward induction based on Arrow-Debreu securities. See for example Hull and White (1994).

4.4. The mortgage prices embodied in Figure 4.4 represent the values immediately *after* the periodic payment has been made. This holds for all figures in this chapter.

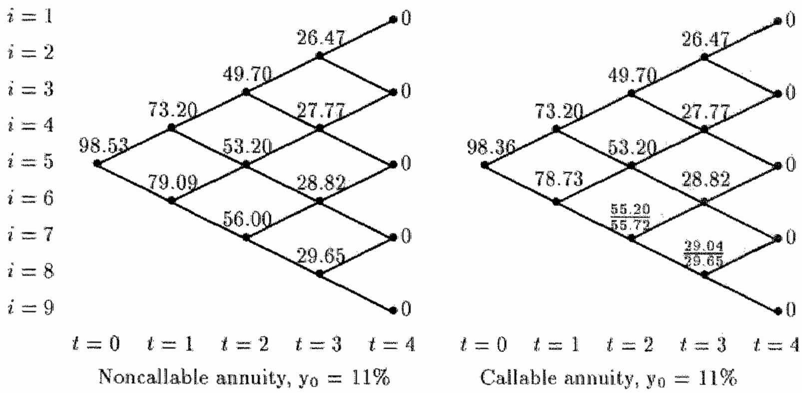


FIGURE 4.4: ANNUITY VALUE

This figure compares the value of callable and noncallable annuities. In the right diagram, prepayment is recommendable in the nodes which contain two values. The lower number represents the present value of the future cash flows. The upper number reflects the unpaid balance of the annuity loan. Since no prepayment costs are included this unpaid balance equals the price at which the annuity can be called.

Annuity-mortgage contracts have the characteristic that each periodical payment consists of both interest payments and principal repayments. Due to these redemptions, the unpaid principal, and hence the interest costs, decrease over time:

$$U_{t+1} = (1 + y_0)U_t - M_0, \tag{4.4}$$

where U_t is the unpaid balance at time t, y_0 is the contract rate at time $t = 0$ which remains constant during the life of the contract, and M_0 represents the periodical costs. The amount which has to be paid in each period remains constant over the life of the contract. These periodical costs are a function of the time to maturity, the size of the principal, and the contract rate:

$$M_0 = U_0 \frac{(1 - v_0)}{v_0(1 - v_0^T)}, \tag{4.5}$$

where U_0 is the initial loan, T is the time to maturity, and $v_0 = \frac{1}{1+y_0}$. Given a contract rate of 11% and an initial loan of 100, the periodical payment is equal to 32.23.

An annuity is a fully amortizing fixed-income security. The loan is completely repaid after four years. Hence, the node values at time $t = 4$ are zero in Figure 4.4 independent of the state of the economy at that time, $V_i(4) = 0$ for all i.

To find the value at any specific node at time $t = 3$, the expected value at $t = 4$, given the node at $t = 3$, has to be raised by the periodical payment paid at $t = 4$. The resulting sum has to be discounted at rate r which corresponds with the node under consideration for time $t = 3$, *e.g.* the value of the noncallable annuity-mortgage at time $t = 3$ in node $i = 8$ is equal to $29.65\left(= \frac{\frac{1}{2}0+\frac{1}{2}0+32.23}{1+0.0872}\right)$. This must be done for all nodes at time $t = 3$. Once these values are known, those at time $t = 2$ can be found. By working all the way back through the tree the value of the security at origin is found. In essence, this is how the backward pricing method works. By doing this for various coupons the rate can be found at which the contract value at origin equals the principal value. For a noncallable annuity this is illustrated in the left diagram of Figure 4.5.

The right diagrams of Figures 4.4 and 4.5 summarize the valuation procedure of a callable fixed-rate annuity-mortgage. Once again a backward technique is utilized. However, instead of feeding values at time t straight into earlier ones, we now have to first check whether exercising the option is preferable to holding the contract. For this the calculated value has to be compared with the exercise price. Rather than staying constant over time, the exercise price is equal to the outstanding debt at each point in time provided no prepayment took place at an earlier stage.

To illustrate, let us once again look at the node corresponding with $i = 8$ at time $t = 3$ in Figure 4.4. As we just saw, the node value is equal to 29.65 when the contract rate is 11%. However, the outstanding debt, and therefore the exercise price, at time $t = 3$ is equal to 29.04. A wealth-maximizing borrower will thus prepay the loan so that the correct value in this node is 29.04 rather than 29.65. This value of 29.04 must be used in the next step of the backward induction procedure. This is illustrated in the right-hand diagram of Figure 4.4. Working back like this through all nodes up until $t = 0$ will bring us to the value at origin.

In the right hand diagram of Figure 4.5 the contract rate is derived which equals the value at origin to the initial loan. The endogenized contract rates at which the callable and noncallable annuities are zero net present value investments differ by 40 basis points in this example.

4.5 Adjustable-rate mortgages

An adjustable-rate mortgage (ARM) is a loan whose contract rate is periodically reset. This periodic adjustment reassigns part of the interest rate risk from the lender to the borrower. The borrower's uncertainty is often partially offset by cap and floor-features embodied in the ARM contract. These features restrict the degree by which the contract rate can fluctuate between reset dates (periodic cap or floor) or during the entire life of

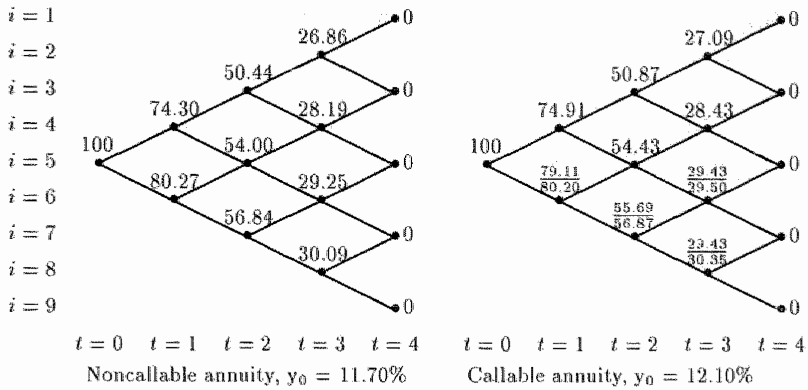


FIGURE 4.5: ANNUITY RATE

the mortgage (lifetime cap or floor). A further explanation of ARM characteristics is given in Chapter 3. In order to illustrate the ARM valuation procedure as introduced by Kau, Keenan, Muller and Epperson (1990, 1993), for the purposes of this section, let us look at an ARM contract both with and without a lifetime cap and floor. Periodic caps and floors are ignored in this four-period example. When the adjustment date is at time $t = 2$, the new contract rate will depend both on the prevailing interest rate at that time and, due to the embedded caps and floors, also on previous contract rates. These earlier rates cannot be known by working backwards through the tree. However, Kau *et al.* (1990, 1993) observe that the value of the promised mortgage payments, the value of the prepayment option and therefore the value of the mortgage are homogeneous of degree one in the unpaid balance. Hence the problem can be solved for an arbitrary unpaid balance and rescaled as required.

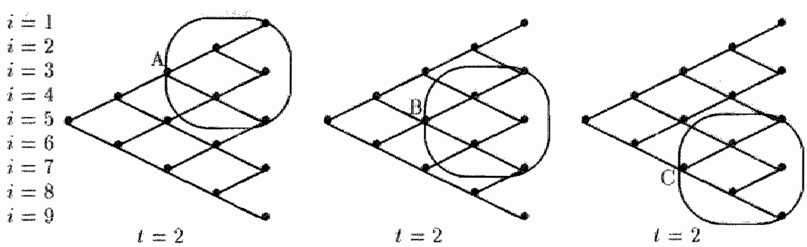


FIGURE 4.6: THE VALUATION OF AN ARM: STEP 1

A slightly adjusted procedure is followed in order to value the payments resulting from an annuity-mortgage whose contract rate is reset at time $t = 2$. Let us start by assuming that the unpaid balance at the adjustment date is equal to 100, which can later be rescaled if necessary. A contract rate is chosen successively for each node in the tree at $t = 2$ such that the value of the annuity-mortgage is equal to one hundred in that node. This is illustrated in Figure 4.6. The sub-tree inside the ellipse of the left-hand diagram determines the contract rate at which the annuity-mortgage is a zero net present value investment for node A at time $t = 2$. (See the previous section.) Once this is done for both node A, B and C, we can continue with the second step in the pricing algorithm as illustrated in Figure 4.7.

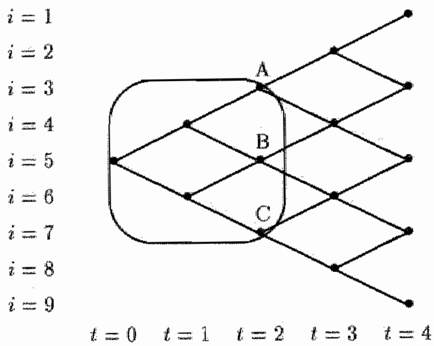
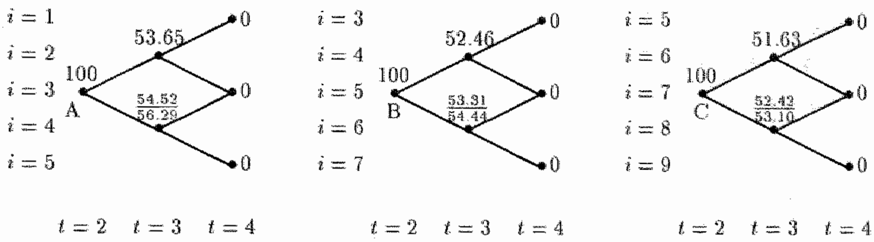


FIGURE 4.7: THE VALUATION OF AN ARM: STEP 2

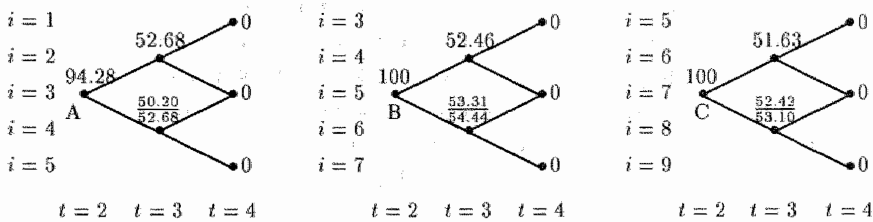
The value of the contract at nodes A, B and C is discounted to $t = 0$. It must be taken into account that prepaying is possible at time $t = 1$. By doing this for various starting contract rates, or using an optimizing algorithm, these two steps will lead to the contract rate at origin which equals the value of the ARM contract to the loan.

Figure 4.8 illustrates the first step for valuing an adjustable-rate annuity-mortgage without cap or floor restrictions. When the new contract rate can freely be reset, the contract value on the adjustment date is at par. This par value is equal to what the outstanding loan would be at that point in time provided no prepayment took place at an earlier stage. It is not necessary to consider the periods after the reset date when valuing an ARM contract without caps and floors. Studying the first two periods and moving directly to the second step will suffice, as summarized in the left diagram of Figure 4.10. However, when cap and floor restrictions are imposed, the first step increases in importance. Now a valuation procedure is needed which can alternate between backward and forward solution techniques. First, the contract rate at $t = 0$ has to be calculated as if no restrictions hold. The reset date must then be analyzed to see whether the cap or floor restrictions become binding. If so, the proposed contract rates in nodes A, B and C need to be adjusted. Next, the contract value at origin must be calculated. The initial contract rate has to be adjusted if this value is not equal to 100 and the whole procedure repeated.



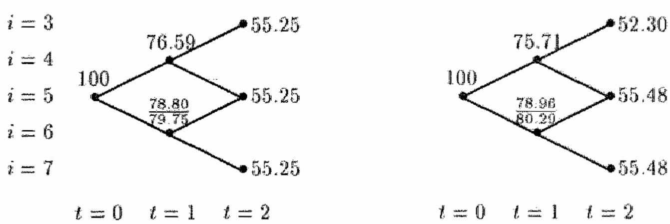
$y_{2,A} = 19.85\%$ $y_{2,B} = 14.20\%$ $y_{2,C} = 10.15\%$

FIGURE 4.8: THE VALUATION OF AN ARM WITHOUT CAP OR FLOOR: STEP 1A



$y_{2,A} = 14.53\%$ $y_{2,B} = 14.20\%$ $y_{2,C} = 10.15\%$

FIGURE 4.9: THE VALUATION OF AN ARM WITH A CAP OR FLOOR: STEP 1B



Without a cap or floor, $y_0 = 11.11\%$ With a cap and floor, $y_0 = 11.63\%$

FIGURE 4.10: THE VALUATION OF AN ARM: STEP 2'

The results of a four period example are illustrated in Figure 4.9 and the right hand diagram of Figure 4.10. In this example, the contract rate is reset after two periods and the cap and floor restrictions require that the rate reset at the adjustment date may not exceed 1.25 times the initial contract rate, while not decreasing beyond 0.75 times the initial rate.

If an initial contract rate of 11.63% is chosen, the cap and floor restriction impose a contract rate reset at $t = 2$ of between 14.53 and 8.73%. If the contract rates could be freely reset at the adjustment date, the new rates would be 19.85, 14.20 and 10.15% for nodes A, B and C, respectively. However, the cap requires that the new contract rate may not exceed 14.53%, so that this latter percentage is now chosen for node A. As Figure 4.9 shows, this results in a below par value for node A. Based on the initial contract rate of 11.63%, the outstanding balance at time $t = 2$ is equal to 55.48. Since the values at node B and C are equal to par, 55.48 is the correct corresponding value for these nodes. Node A value is found by rescaling ($52.30 = 94.28\% \times 55.48$). The next step is now to discount these values plus the remaining cash flows (once again keeping prepayment opportunities in mind) for time $t = 0$, the explanation for which is given in Section 4.4.

It is evident that this procedure is computationally intensive. This imposes serious restrictions when a more realistic setting is chosen, such as a longer time to maturity, more than one reset date with periodic caps and floors, or a multi-factor interest rate model which requires a much larger and finer interest rate grid. For example, Kau *et al.* (1990) utilize a 64 point grid for the interest rate and a 21 point grid for the lagged contract rate. In their 1993 paper which introduces the concept of default, an interest rate grid of 24 points is used to span the range of possible contract rates. Values lying in between these grid points are found by simple interpolation. This illustrates once again the computational restrictions stressed in Section 4.2. By using a sophisticated model to link the mortgage rate to the short-term interest rate, Kau *et al.* (1990, 1993) must make do with a small grid in their numerical solution procedure.

4.6 Mortgages with limited prepayment options

Most Dutch mortgages cannot be fully called without a penalty. Only 10 or 20 percent of the initial loan can commonly be prepaid in a calendar year without surcharge. Additional prepayments are settled at a cost which is equal to the present value of the difference between the future monthly payments of a new contract and the existing mortgage. Due to the limited prepayment restriction, the path-dependence problem is even more profound in valuing these types of contracts than it was in the previously discussed adjustable-rate mortgages. Now the mortgage loan is no longer either fully prepaid or fully outstanding, various in-between stages are possible as well.

When a mortgage is partially prepaid, the periodical payments of the borrower must be adjusted for the smaller outstanding debt. The contract rate does not alter as a result of the prepayment. The newly determined periodical costs are therefore a function of the remaining outstanding debt and the remaining time to maturity. For a fully amortizing annuity-mortgage it thus makes a substantial difference whether the mortgagor exercises the option to partially prepay the loan in January or in December. This extra path-dependency makes a backward pricing technique very difficult. Due to the computational complexity, a valuation technique which alternates between the backward and forward solution methods is also hard to implement. Figures 4.11 and 4.12 illustrate a four-period example. Even a small example like this shows the difficulty involved in valuing this contract by using interest rate tree methods.[5]

Figure 4.11 plots a tree which illustrates the cash flows corresponding with a contract which will be partially prepaid whenever the prevailing interest rate is lower than the interest rate at origination. The time steps in Figure 4.11 represent calendar years. The month of prepayment is thus ignored in this example. In Figure 4.11, M represents the periodical payment and the first subscript indicates when the size of this payment was determined. Successive subscripts illustrate the path the interest rate takes up to the node where the mortgage was partially prepaid and a new periodic payment arranged. Once again u means that the interest rate went up and d illustrates a downward movement. Looking at the difference between $M_{3,udd}$ and $M_{3,ddu}$ will help clarify the notation. Both periodical payments are determined at time $t = 3$. However, the first follows from interest rates which first increased and then decreased twice in a row. Following this interest rate path, the first partial prepayment will be at $t = 3$. The other periodical payment, $M_{3,ddu}$, is based on an interest rate path which first went down twice in a row before rising again. So at time $t = 1$ and $t = 2$ the mortgage will also be partially prepaid. Following this latter interest rate path therefore results in a smaller periodical payment settled at time $t = 3$ than when the first path is followed. This holds even though at time $t = 3$ the same interest rate node is reached.

The variable b in Figure 4.11 illustrates that when the interest rate path reaches the corresponding node, the mortgage will be partially prepaid. These nodes are indicated by a circle. The amount indicated by variable b lies between zero and B, where B is equal to the maximum that can be prepaid without cost. As long as the outstanding loan U_t, is larger than B, the borrower will always prepay the maximum B. If the remaining debt is smaller than the allowed prepayment, the borrower will naturally only repay the outstanding debt, so that $b = \min(B, U_t)$. Hence, if the mortgage is completely prepaid in an earlier stage, b will be equal to zero. The same holds true for M: if the contract is completely prepaid then M becomes zero from that node on.

In Figure 4.12 a numerical example is introduced. The contract valued in Figure 4.12 is

[5] Just before this thesis went to publication, research being conducted by Cheuk and Vorst (1997) and Pelsser and Vorst (1997) regarding multiple-shout floors and flexible caps was pending. In both instruments the holder has the right to exercise the derivative when it is in-the-money or to wait for another future situation which might be more profitable. These derivatives are priced with a numerical method which in essence comes down to piling up interest rate trees. This layered approach may shed a new light on the valuation of mortgages with annual prepayment restrictions.

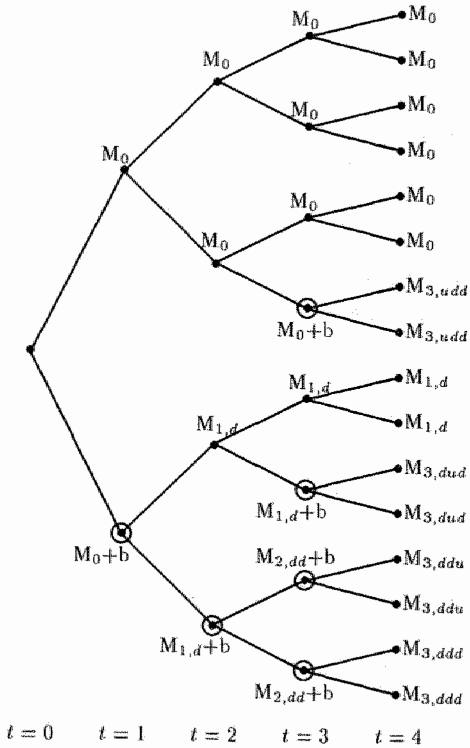


FIGURE 4.11: CASH FLOW TREE

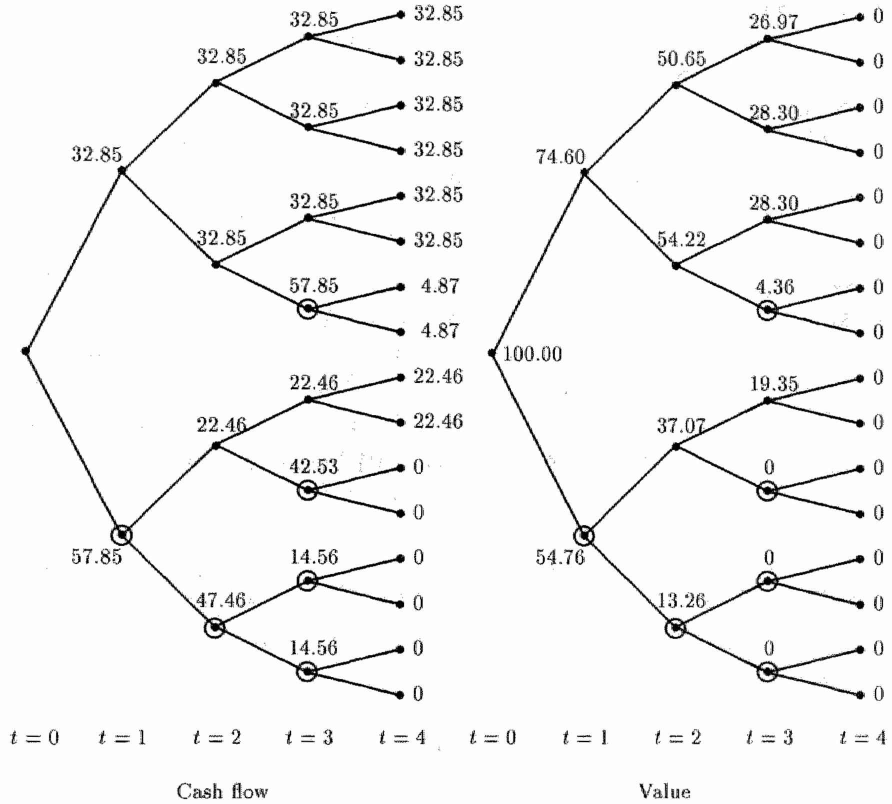


FIGURE 4.12: CASH FLOW AND VALUE TREE, $Y_0 = 11.89\%$

an annuity mortgage loan of 100, with a contract rate of 11.89% and a maturity of 4-years. Only 25% of the initial loan is allowed to be prepaid each year. As long as the outstanding loan is larger than zero, the borrower will partially prepay the loan whenever the prevailing interest rate is lower than the interest rate at origin.

The left diagram of Figure 4.12 shows the cash flows for each node following from this contract. In the right diagram, the stream of possible payments is valued.

Let us first look at the tree illustrating the cash flows. By using the aforementioned numbers, in Equation (4.5) the periodical payment determined at origin, M_0, is equal to 32.85. When the interest rate goes up in the first period, the cash flow at time $t = 1$ will be equal to 32.85. In the branches proceeding from this *up*-node at $t = 1$, only one node is found where the contract will partially be prepaid. Given the binomial tree, this node can be reached at time $t = 3$ when the outstanding balance is equal to 29.36. Since the borrower can only prepay 25% of the initial loan, the cash flow in that node is equal to 57.85 (= 32.85 + 25). The debt that remains is equal to 4.36, the remaining time to maturity is 1 year and the contract rate remains 11.89%. It thus follows from Equation (4.5) that the periodical payment, $M_{3,udd}$, is 4.87. This results in the cash flow in the nodes caused by the sprouting branches.

Instead of moving upwards, the interest rate can also decrease in the first period. This will immediately result in partial prepayment such that the cash flow in the *down*-node at $t = 1$ is equal to 57.85. With no prepayment the outstanding debt after one period would be 79.04. However, after the partial prepayment it becomes 54.04 so that the periodically required payment can be reduced to 22.46. From this *down*-node at $t = 1$ the interest rate can move either up or down. In the former case no partial prepayment will occur, the cash flow is 22.46, and the outstanding debt equals 38.01. Continuing on from this node, the cash flow in the next period will equal either 22.46 or 42.53. The latter cash flow consists of the periodical payment of 22.46 plus the partial prepayment. Since the outstanding debt is now only 20.07, the borrower will repay exactly that amount. The mortgage being completely repaid, the cash flows in nodes on branches that proceed from here are now equal to zero.

A similar approach should be followed in the event the interest rate decreases in both the first and second period, so that the cash flow tree on the left hand side of Figure 4.12 can be completed. The cash flows shown in this tree were found by going forward through the tree. The next step is to discount the cash flows by going backwards through the tree. This is illustrated by the right hand diagram of Figure 4.12.

4.7 Conclusion

The principles of mortgage valuation are explained in this chapter with the help of simplified numerical examples based on bond option pricing techniques. The starting point is a description of the interest rate dynamics, which is assumed to be the single fundamental variable which drives all other interest rates. This assumption simplifies the description of the term structure of interest rates and its dynamics. In this chapter, the Black, Derman and Toy (1990) interest rate model is applied. Given a fictitious term structure of interest rates, this model constructs a corresponding interest rate tree which is used for the examples included in this chapter.

First, noncallable and callable fixed-rate mortgage-annuities are valued. These loan types can be valued with the techniques developed for bond options. It becomes more difficult when noncallable and callable adjustable-rate mortgages are priced. Kau, Keenan, Muller and Epperson (1990, 1993) introduced a pricing technique to value these kinds of mortgage contracts. Their technique is computationally intensive and therefore requires sacrifices in the accuracy of the interest rate process. In the last section a typical Dutch mortgage contract is studied, for which only a limited percentage of the initial loan can be annually prepaid. Due to the resulting path-dependency of the cash flows, this loan type cannot be valued by backward induction only. Theoretically, a valuation method which alternates between the backward and forward solution technique as applied by the valuation of ARMs could solve this valuation problem. However, even more than is the case for ARMs, applying this method here causes even small examples to become unmanageable. Introducing an exogenous prepayment rule, as was done in this chapter, reduces the problem. In this case a forward solution technique can be applied to construct an interest rate and cash flow tree. Various things need to be known at each node: is prepaying recommendable, is it still allowed or has another prepayment occurred earlier in the same calendar year, what is the required periodical payment, and what is the outstanding debt? Only the first question is node-specific, the other three depend on the path of the interest rate. Due to this path-dependency, a recombining tree no longer suffices. However, the number of nodes in a non-recombining tree increases rapidly. When a binomial lattice is utilized the number of nodes double with each step. Consequently, introducing an exogenous prepayment rule is insufficient for addressing realistic valuation problems. For this we have to resort to simulation techniques as done in Chapter 7.

Chapter 5

Mortgage pricing under alternative prepayment behavior

5.1 Introduction

The main difficulty in pricing mortgages lies within the prepayment behavior of the mortgagor. The literature distinguishes between optimal prepayment and exogenous prepayment rules. Under optimal prepayments the valuation proceeds as for any callable bond by starting at the maturity date of the contract and working backwards in time under the assumption that, at each point in time, the borrower prepays when the value of the mortgage, if left uncalled, exceeds the outstanding debt plus any transaction costs associated with refinancing it.

Exogenous prepayment rules are empirically determined and based on observable variables like the age of the loan and the refinance incentive.[1] As described in Chapters 3 and 9, this refinance incentive captures the homeowners' decision to refinance their mortgage if the prevailing mortgage rates are lower than the coupon rate on their contract. This prepayment behavior can differ from the optimal call policy.

In this chapter we compare both approaches and examine how both prepayment rules affect the pricing of mortgages. First a valuation model is developed which ensures optimal prepayment behavior. Subsequently, a model is constructed in which the borrower prepays as soon as this reduces the future costs. Both approaches yield identical prepayment behavior as long as there are no market imperfections. As Kau and Keenan (1995) state: "*If there were no up-front points or insurance, borrowers would prepay as soon as interest rates fell*". However, each approach might yield different prepayment behavior if market imperfections are introduced. In this chapter we concentrate on the impact transaction costs have on prepayment behavior and study the resulting differences of both approaches.

The optimal prepayment rule assumes that the mortgage market is a highly liquid and competitive market on which no intermediary has sufficient market power to charge

[1] Examples are Richard and Roll (1989), Kang and Zenios (1992) and Golub and Pohlman (1994).

contract rates which exceed its own cost of capital. This allows us to value a mortgage with no-arbitrage principles. For this we must impose that the mortgage is a zero net present value investment, *i.e.* the value of a mortgage at origination has to equal the face value of the loan. To ensure that this holds, we have to endogenously derive the relation between the mortgage rate and other interest rates. Such a model may otherwise prescribe prepayment at moments when the prevailing mortgage rate exceeds the contract rate on the current loan.

The prepayment rule which prescribes prepayment when the prepayment option is in-the-money depends explicitly on the mortgage rate at which the homeowner can refinance his loan in the market. Whether this mortgage rate is endogenously or exogenously determined is not important. This approach automatically guarantees that prepayment only occurs if the prevailing mortgage rate is less than the current contract rate. As a shortcut to valuing mortgages we can therefore use empirically observed relations between mortgage rates and interest rates. In this chapter, for example, we use the historical relation between the mortgage rate and the short-term interest rate in the Netherlands and the US.

The main advantage of an exogenous relation between the mortgage rate and the short-term interest rate is that it allows valuation models to cope with complicated path-dependency problems, detailed prepayment restrictions and complex interest rate environments. For example, the prepayment rule applied in this chapter is also utilized in Chapter 7, where the annual prepayment restrictions are analyzed together with the minimum interest rate guarantee frequently embodied in the quotation offer preceding a Dutch mortgage contract. Within that chapter, a multi-factor interest rate model is used. Since the same forward-looking prepayment rule is used, we can compare the valuation results of Chapter 7 with the results proceeding from the valuation method of this chapter. A major limitation of this approach is that the exogenously defined prepayment rule does not automatically lead to optimal prepayment behavior in each theoretically possible setting. Before we apply this prepayment rule in the simulation model we must therefore first examine its strengths and weaknesses. In this chapter we conduct an extensive analysis of the valuation model based on this rule and compare its results with those of a model which is based on the optimal prepayment rule.

The second topic that is central to this chapter is the impact interest rate processes have on mortgage pricing. Since a mortgage contract is an interest rate derivative with many option features, its valuation and risk measures could be very sensitive to the dynamic process of interest rates. For this reason we compare the results of three alternative endogenous single factor interest rate models, all estimated on Dutch short-term interest rate data.

The remainder of the chapter is organized as follows. Section 5.2 describes the mortgage contract and introduces the relevant components of an accurate mortgage valuation model. Section 5.3 elaborates on the valuation procedure and Section 5.4 studies the interest rate risk of a mortgage. The interest rate data used in this paper are summarized in Section 5.5. Section 5.6 contains an empirical analysis of the time series process of the short-term interest rate. Section 5.7 discusses the empirical relation between the mortgage rate and the short-term interest rate. The valuation results based on the optimal prepayment rule follow in Section 5.8, while Section 5.9 presents the results from the suboptimal rule. Section 5.10 concludes.

5.2 Mortgage valuation

Many Dutch mortgages are annuities that pay a constant monthly cash flow consisting of both interest and redemption. The most popular mortgage has a maturity of thirty years with the interest rate fixed for a five year period. After each five year period the contract rate is reset. No cap or floor restrictions apply on setting the new contract rate at the adjustment date. In the mortgage valuation models we therefore only consider the single fixed rate period. Interest rate risk faced by the lenders does not extend to periods after this adjustment.

A mortgage contract contains multiple embedded options. Most important is the prepayment option which allows the borrower to call the contract. We will consider a situation where the complete mortgage can be prepaid. However, prepayment is discouraged by the up-front fee for starting a new mortgage contract, necessary to refinance the existing contract. These costs are incurred by the borrower but not necessarily received by the investor.

A mortgage valuation model consists of four major components: a model of the term structure of interest rates, a model of interest rate dynamics, the relationship between mortgage rates and the term structure, and a prepayment behavior model.

5.2.1 Term structure of interest rates

In essence there are two approaches to model the term structure of interest rates, both of which are touched upon in Section 3.2. The *general equilibrium approach* starts by describing the underlying economy, the stochastic process of one or more exogenous state variables and the preferences of a typical investor. The term structure of interest rates is endogenously derived based on these elements, hence the term *endogenous term structure models*. The most famous model in this category is the Cox, Ingersoll and Ross model (1985a,b).

No-arbitrage models in the second approach take the initial yield curve as given and derive feasible subsequent term structure movements consistent with no-arbitrage opportunities. Examples include the Ho and Lee model (1986), the Black, Derman and Toy model (1990), and the model developed by Heath, Jarrow and Morton (1990, 1992). These models are often referred to as *exogenous term structure models*, because they take the term structure of interest rates as an input rather than producing it as an output.

The strength of the exogenous approach is that it utilizes the full information of the term structure observed on the valuation date. However, today's term structure is fitted at the expense of describing the general interest rate dynamics. Moreover, if the term structure changes the next day, the interest rate diffusion process has to be readapted accordingly. In other words, the interest rate dynamics may change from day to day. In contrast, the parameters of endogenous models are assumed to be time-homogeneous. These endogenous models describe general interest rate dynamics while no reference is made to the observed term structure at a specific valuation date. Ideally, the endogenous models should also fit today's term structure but this is not explicitly imposed.

The type of application determines which models are appropriate. Exogenous models, for example, are preferable in determining today's value of a mortgage contract, while the time-invariant parameters of endogenous models make these models more suitable for analyzing the risk characteristics of a contract. This chapter focuses on the consequences of loosening prepayment restrictions. A bank which considers issuing freely callable mortgages in the near future will be interested in the risk characteristics of these loans. Since these mortgages will not be issued today, there is no reason to force the interest rate dynamics to be consistent with today's term structure. Instead, a more accurate description of the general interest rate dynamics is desirable. In this chapter we therefore utilize endogenous interest rate models to describe the term structure of interest rates.

5.2.2 Short-term interest rates

In this chapter we assume that the interest rate dynamics can be modelled by a one-factor interest rate process. Most of the mortgage valuation literature uses the Cox, Ingersoll and Ross (CIR, 1985a,b) square root process for this. However, this model does not correspond well with the actual dynamics of the short-term interest rate. The volatility of interest rates is much more sensitive to the level of interest rates than in the CIR model. In addition, the drift is nonlinear, being almost zero at low interest rate levels and negative at high levels.[2] Therefore, in this chapter we do not limit ourselves to the Cox, Ingersoll and Ross model, but also use a nonlinear and a nonparametric model to describe the short-term interest rate dynamics.

These dynamics affect the value of the mortgage in three ways. First, they determine the term structure of interest rates and thus the discount factors of the periodic cash flows. Second, in an endogenous mortgage rate model the short-term interest rate process implies a dynamic process for the mortgage rate. Third, they affect the prepayment behavior.

In an endogenous one-factor model the distribution of next month's interest rate depends only on today's interest rate. If the parameters of the empirical interest rate model are time invariant, these interest rate processes are all Markovian. For computational reasons we approximate the different interest processes by a discrete time Markov chain with a finite number of states i, $(i = 1, \dots N)$.[3] A Markov chain can easily describe nonlinear

[2] See for example Chan, Karolyi, Longstaff and Sanders (1992), Aït-Sahalia (1996a,b), Pfann, Schotman and Tschernig (1996), Tauchen (1996), Koedijk, Nissen, Schotman and Wolff (1997), and Conley, Hansen, Luttmer and Scheinkman (1997).

[3] The theory of Markov chains can be found in many textbooks on stochastic processes, *e.g.* Ross (1993).

time series behavior of the short-term interest rate. Details of the empirical spot rate processes are discussed in Section 5.6.

5.2.3 Mortgage rates

The model which relates the short-term interest rate dynamics to the mortgage rate determines the monthly payments and affects the opportunity costs for borrowers who consider refinancing. It therefore influences the prepayment behavior in the forward-looking valuation model.

In a single factor model, the short-term interest rate process involves the only uncertainty in the economy. The state of the economy depends therefore only on the level of the short-term interest rate. Each state corresponds with only one mortgage rate, such that the one-factor assumption implies that there is a one-on-one relationship between the short-term interest rate r and mortgage rate y.

We consider three specifications to relate the mortgage rate to the short-term interest rate. Following Kau, Keenan, Muller and Epperson (1993) we derive two functional relations $y = f(r)$ endogenously for both prepayment rules applied in this chapter. Hereby the mortgage rate is determined such that the value of the mortgage contract at origination is equal to the face value of the loan. As discussed in Section 5.2.4, in this setting a mortgage is an interest rate derivative security with zero net present value. To derive the endogenous relation it is assumed that mortgages are traded in a competitive market.

Alongside the endogenous relations, we also consider two exogenous specifications between the mortgage rate and the short-term interest rate. The first is the empirical relation between the Dutch mortgage rate and the Dutch short-term interest rate. An advantage of this approach is that it is directly related to Dutch historical interest rate data. Despite this empirical justification, this relation is subject to the Lucas critique, as described in the introductory chapter, *i.e.*, this relation describes the actual historical situation where prepayment was very much restricted. Commonly, only 10 to 20 percent of the initial loan could be called within a calendar year without penalty. Due to the increased competition, Dutch financial institutions are currently considering the consequences of loosening these annual prepayment restrictions. In this chapter all prepayment restrictions are omitted such that the mortgage can be fully called. The Dutch historical premium is probably too small to cover this additional prepayment risk. Therefore, the second specification considers the empirical relation between the US mortgage rate and the US short-term interest rate. In the US, annual prepayment limitations are virtually unknown. Consequently, the US relation shows a higher option premium, but depends on the US term structure, which can differ from the Dutch term structure. Section 5.7 provides details of the alternative empirical relations between the mortgage rate y and the short-term interest rate r.

5.2.4 Prepayment behavior

The valuation models in this chapter closely follow the setup of Dunn and McConnell (1981a,b) and McConnell and Singh (1994). Dunn and McConnell (1981a,b) assume that the mortgage market is frictionless and that borrowers prepay their mortgage as soon as the value of the mortgage, if left uncalled, exceeds the unpaid balance of the loan plus the transaction costs associated with refinancing it. McConnell and Singh (1994) extend this valuation model and incorporate transaction costs into their rational valuation framework. At each point in time they determine the critical interest rate below which it is rational to prepay the mortgage. The difficulty is in determining this boundary. McConnell and Singh's derivation of the boundary depends on the crucial assumptions that (a) the mortgagor can borrow money for the same costs as the mortgagee, and (b) the competition on the mortgage market is strong enough to ensure that lenders cannot charge a mortgage rate that exceeds their own cost of capital. If the latter assumption does not hold, it is possible that their prepayment rule prescribes prepayment even though the mortgage rate at which the borrower can refinance is higher than the contract rate of the current loan.

To see this, let us go back to the examples in Chapter 4 where the backward valuation technique was explained. As before, a borrower is assumed to prepay the loan when the present value of the loan exceeds the outstanding debt plus any transaction costs. An important property of this prepayment rule is that it is entirely determined by the spot rate process, without any reference to refinance alternatives open to the borrower, *i.e.* there is no rôle for the mortgage rate at which the homeowner can obtain a new mortgage loan.

Suppose that shortly after a mortgage contract is issued, the short-term interest rate decreases but that the intermediary has sufficient market power to keep the mortgage rate at the old level. The decrease in the short-term interest rate causes the present value of the mortgage to rise. If this rise is large enough to trigger prepayment, the borrower will replace the existing contract by taking out a new loan and paying the corresponding transaction costs. Despite these costs, the borrower is facing the same contract rate as before! This irrational prepayment behavior is excluded if the mortgage rate is endogenously determined as a function of the short-term interest rate, such that the value of the new mortgage at its origination is equal to the principal of the loan. Exogenous relations between the mortgage rate and the short-term interest rate do not have to be consistent with this and can therefore not be incorporated in this framework.

In this chapter we relate the optimal prepayment rule to a suboptimal one which does not require the assumption that the mortgage has a zero net present value at origination. The suboptimal prepayment rule applied here assumes that the mortgage is replaced by a new contract if this reduces the total future costs for the borrower. Hence, the call policy depends explicitly on the mortgage rate at which the borrower can refinance in the market. This is in sharp contrast to the optimal prepayment rule which depends only on the spot rate process. The suboptimal prepayment rule allows the financial markets to be segmented such that the borrower does not have direct access to the capital market and can only obtain the funds to finance the purchase of a house through an intermediary. This intermediary may have sufficient market power to charge an interest rate which exceeds its own cost of capital.

For this suboptimal prepayment rule we calculate a critical mortgage rate $y_c(t)$, for each month t, below which refinancing would reduce the future costs for the borrower.[4] In computing the alternative costs we assume that the new mortgage loan is for the remaining time to maturity. To start a new mortgage, the borrower has to pay up-front costs equal to c percent of the unpaid balance U_t at time t. These costs discourage prepayment, and imply that the mortgage rate has to fall sufficiently below the initial contract rate. The refinancing costs can be interpreted as a premium on the unpaid balance U_t. Technically, the mortgage is callable at $(1+c)$ times its par value. Hence, a mortgagor who decides to replace the existing contract has to borrow $(1+c)U_t$ at the new mortgage rate. The future monthly costs M_t^* of the new loan are:

$$\begin{aligned} M_t^* &= (1+c)U_t\frac{(1-v_t)}{v_t(1-v_t^{T-t})}, \\ &= (1+c)M_t, \end{aligned} \tag{5.1}$$

where $v_t = \frac{1}{1+y(t)}$, with $y(t)$ representing the prevailing mortgage rate at time t. Since T is the maturity of the original contract, $(T-t)$ reflects the remaining time to maturity in months. M_t are the monthly payments resulting from an annuity with maturity $(T-t)$ issued at time t without refinancing costs, M_t^* does include these costs.

The borrower can also decide to retain the original mortgage in which case the monthly costs remain:

$$\begin{aligned} M_0 &= U_0\frac{(1-v_0)}{v_0(1-v_0^{T})}, \\ &= U_t\frac{(1-v_0)}{v_0(1-v_0^{T-t})}. \end{aligned} \tag{5.2}$$

Hence, if the fixed-rate period is equal to the lifetime of the contract the prepayment rule reads:

$$(1+c)M_t < M_0. \tag{5.3}$$

However, when the fixed-rate period is shorter than the time to maturity, this prepayment rule is a necessary, but no longer sufficient condition to ensure that refinancing results in lower future costs. For example, suppose that one month before the fixed-rate period ends, Inequality (5.3) holds. Would you refinance the contract and pay the transaction

[4] In order to avoid confusion regarding the states of the economy, the time demarcations for mortgage rate y, and short-term interest rate r are parenthesized as opposed to subscripted, *i.e.*, $r(t)$ represents the short-term interest rate at time t, while r_i indicates the short-term interest rate corresponding with state i. For all other variables the time demarcation remains subscripted.

costs? Or would you wait one more month and get the contract rate adjusted to the then prevailing mortgage rate for free? You will probably only replace your mortgage if the resulting savings on the periodical payment exceed the transaction costs. Or more generally, homeowners will only replace their mortgage if the resulting savings on their periodical payments during the fixed-rate period justify the transaction costs. To determine this, we compare the outstanding debt on the adjustment date of an uncalled mortgage with the outstanding debt of the new mortgage loan on that reset date. With mortgage rate $y(t)$ prevailing at time t and the corresponding new periodic cash flow M_t^*, the unpaid balance on the adjustment date τ of a new contract issued at time t is:

$$\tilde{U}_\tau = \frac{U_t(1+c)}{1-\left(\frac{1}{1+y(t)}\right)^{T-t}} \times \left[1 - \frac{(1+y(t))^{\tau-t}}{(1+y(t))^{T-t}}\right], \tag{5.4}$$

with $t < \tau < T$. This outstanding debt is compared with the debt on that same date if the mortgage is not called:

$$U_\tau = \frac{U_t}{1-\left(\frac{1}{1+y(0)}\right)^{T-t}} \times \left[1 - \frac{(1+y(0))^{\tau-t}}{(1+y(0))^{T-t}}\right]. \tag{5.5}$$

Let us look at a mortgagor who replaces the contract at time t with a new loan which demands lower monthly payments. Each month, until the reset date, the individual saves $(M_0 - M_t^*)$. If the homeowner deposits these savings each month on a bank account on which he earns interest, he will have an amount S_τ at his disposal at time τ.

Due to the refinancing costs the debt initially increases. On the adjustment date τ, the outstanding debt of the new loan can therefore be larger than what the debt would be if the contract is left uncalled. A borrower is indifferent between refinancing the contract and holding on to it if S_τ, the present value of the savings at time τ, is large enough to offset the potential higher debt on the reset date.[5] Mathematically, the prepayment option is in-the-money if:

$$\tilde{U}_\tau - S_\tau < U_\tau, \tag{5.6}$$

where S_τ is the value of all monthly savings expressed in dollars of time τ. To calculate S_τ we must use the short-term interest rates between time t and τ. However, these rates are unknown at time t when the prepayment decision is made. Given the state of the economy at time t, we can use the interest rate grid and Markov transition probabilities to calculate the expected short-term interest rate for each month. Hence, the expected value of S_τ can be determined.

The critical mortgage rate $y_c(t)$ at which $\tilde{U}_\tau - S_\tau = U_\tau$ defines an upper boundary for the mortgage rate above which prepayment will not occur. This boundary depends on both the underlying process of the interest rate dynamics and the model which relates the

[5] A typical Dutch mortgage can be prepaid without a penalty on the reset date such that S_τ can be used to reduce the unpaid balance at time τ.

short-term interest rates to the mortgage rate. If $\tilde{U}_\tau - S_\tau < U_\tau$ refinancing reduces the future costs from time τ onwards due to the fact that the new periodic payments are based on a lower principal amount at the reset date.

The nature of the prepayment rule is such that it may prescribe prepayment during stages when waiting may have been more profitable. However, the value of the prepayment option diminishes with the lapse of time, ceteris paribus. At maturity this option value is, with certainty, equal to zero. In the Netherlands where the contract rate is freely reset after a fixed-rate period, the time value of the prepayment option vaporizes even faster. A borrower who decides to wait to prepay the mortgage is weighing this decrease in value against the possible gain that can be achieved by postponing to exercise the prepayment option. However, this decision to wait is not without costs. Not only will the option value decrease with the lapse of time, the mortgagor is also saddled with higher monthly costs during the time he is waiting for a possible decrease in the mortgage rate. For these reasons we do not expect large differences between the optimal and suboptimal prepayment rule.

5.3 Valuation procedure

In Chapter 4 an interest rate tree was constructed to model the dynamics of a one-factor model. There, a binomial tree was used for illustrative purposes. In this chapter we apply a multinomial recombining tree which immediately branches out to its full size. The main advantage of such a multinomial interest rate tree is that it can capture the characteristics of more general interest rate processes without the number of nodes growing exponentially with the number of time steps.[6] The transition probabilities of the recombining interest rate tree are stored in the Markov transition matrix $\mathbf{A}$. Each element a_{ij} of this transition matrix contains the probability that the interest rate process moves from state i at time t to state j at time $t+1$, conditional on the interest rate process being in state i at time t. Mathematically that is, $a_{ij} = Pr(z_{t+1} = j \mid z_t = i)$, where z_t is the state of the economy at time t.

From the empirical time series analysis of the short-term interest rate we obtain the actual matrix $\mathbf{A}$ of transition probabilities. However, to value a mortgage contract we have to use risk-neutral probabilities associated with the interest rate process, and not the actual transition probabilities. This requires an assumption about the market price of risk, which can be derived implicitly from the term structure of interest rates.[7] We assume that the spot rate follows a stationary process which implies that the infinite horizon yield converges to a constant R_∞ independent of both time and the current spot rate $r(t)$. Setting R_∞ at a particular value, say $R_\infty = 8\%$, we can solve for the market price of risk and thereby obtain the risk-neutral Markov transition matrix $\tilde{\mathbf{A}}$.

To do this, the elements in each row of $\mathbf{A}$ are shifted to the left (lower states, higher

[6] See Hull and White (1990, 1993) and Nelson and Ramaswamy (1990) for an elaborated discussion on this topic.

[7] See Duffie, Chapter 3, 1996.

interest rates) by δ_i positions. The open positions at the right of each row are filled with zeros and the entire row is rescaled such that the probabilities on row i sum to one.[8] The resulting transition distributions differ from the original vectors of transition probabilities only by their drift. All other aspects of the distribution are preserved as much as possible. The numbers δ_i are obtained by trial and error, such that the implied infinite horizon yield R_∞ equals 8% irrespective of the initial spot rate. For this we define an $(N \times 1)$ vector $\mathbf{\Lambda}_\tau$ with an element $\Lambda_{i\tau}$ being the price of a zero-coupon bond with maturity τ when the spot rate is equal to r_i. These bond prices can be computed as:

$$\mathbf{\Lambda}_\tau = \mathbf{B}^\tau \boldsymbol{\iota}, \tag{5.7}$$

where $\boldsymbol{\iota}$ is a vector of ones and the elements in the matrix $\mathbf{B}$ are discounted risk-neutral probabilities: $b_{ij} = \frac{\tilde{a}_{ij}}{(1+r_i)}$. The vector of yields corresponding with the bond prices has elements $R_{i\tau} = \Lambda_{i\tau}^{-\frac{1}{\tau}} - 1$. We determine the shift parameters δ_i such that

$$\lim_{\tau \to \infty} (\Lambda_{i\tau}^{-\frac{1}{\tau}} - 1) = R_\infty = 8\%, \tag{5.8}$$

for all $i = 1, 2, \ldots, N$. Equations (5.8) are solved simultaneously for all i.

For the special case that the interest rate dynamics are modelled as the CIR square root process the transformation from the actual to the risk-neutral probability measure can be performed analytically. See Section 5.6.1 below.

5.3.1 Optimal prepayments

Optimal prepayment rules lead to dynamic programming problems which must be solved backwards. We start at the end of a valuation tree and work back through all nodes until the value of the contract at the moment of origination is found. In each node of the valuation tree, the mortgagor checks whether replacing the mortgage is preferable to holding the contract. In order to determine this, the present value of the future cash flows of the existing contract is compared with the principal value of the new loan the borrower has to take out to prepay the existing one. Starting a new contract implies incurring new up-front costs, such that the prepayment decision is not just based on the unpaid balance of the old contract at that time, U_t, but also on the refinance costs, c. These costs are commonly proportional to the unpaid balance. Consequently, the mortgage will be prepaid if $U_t(1+c)$ is smaller than the present value of the existing contract. If this is the case in a particular node of the valuation tree, the value fed into the corresponding node becomes $U_t(1+c)$ rather than the present value of future payments. The mortgagee, however, receives only U_t and not $U_t(1+c)$. The value of the matching node in the mortgagee's valuation tree is therefore equal to U_t.

We introduce the following notations to value the contract studied in this chapter:

[8] The amount of shift is always small, so that more than 2.5% of the original probability mass is never pushed out of the matrix at the left-hand side. Since the number of grid points N is large ($N = 226$), the transition densities are only minimally disturbed.

N = number of states,

$\mathbf{v}_t^*$ = a $(N \times 1)$ vector containing the mortgage value for the *mortgagor* at time t before the prepayment decision is made,

$\mathbf{v}_t$ = a $(N \times 1)$ vector containing the mortgage value for the *mortgagor* at time t after the prepayment decision is made,

$\mathbf{w}_t^*$ = a $(N \times 1)$ vector containing the mortgage value for the *mortgagee* at time t before the prepayment decision is made,

$\mathbf{w}_t$ = a $(N \times 1)$ vector containing the mortgage value for the *mortgagee* at time t after the prepayment decision is made.

The mortgage contract studied in this chapter has a maturity of thirty years, and an interest rate fixed for a 60 month period. The backward pricing method therefore starts in the interest rate tree at time $t = 60$.[9] The value at time $t = 60$ equals U_{60} with certainty. At the adjustment date, the mortgagor can freely reset the contract, such that this mortgage value holds for both the mortgagor and the mortgagee. Hence, the elements in both $\mathbf{v}_{60}$ and $\mathbf{w}_{60}$ are equal to U_{60}.

Given $\mathbf{v}_{60}$ and $\mathbf{w}_{60}$ we can calculate the present value of future payments one period earlier by using the matrix $\mathbf{B}$, which contains the *risk-adjusted discounted transition probabilities* as derived before. More generally, to derive $\mathbf{v}_t$ and $\mathbf{w}_t$ given $\mathbf{v}_{t+1}$ and $\mathbf{w}_{t+1}$, we must first determine the mortgage value for the mortgagor at time t *before* the prepayment decision is made:

$$\mathbf{v}_t^* = \mathbf{B}\left(\mathbf{v}_{t+1} + \boldsymbol{\iota} M_0\right), \tag{5.9}$$

where $\boldsymbol{\iota}$ is a vector of ones and M_0 is the monthly required payment. Each element v_{jt}^* is compared with $U_t(1+c)$ and the mortgage is prepaid if $U_t(1+c) < v_{jt}^*$. This prepayment behavior at time t is captured by the diagonal matrix $\boldsymbol{\varphi}_t$ where the j^{th} element on the diagonal position is equal to:

$$\varphi_{jt} = \begin{cases} 1 & \text{if} \quad U_t(1+c) < v_{jt}^*, \\ 0 & \text{if} \quad U_t(1+c) \geq v_{jt}^*. \end{cases} \tag{5.10}$$

The contract value at time t for the mortgagor can now be calculated by:

$$\mathbf{v}_t = \mathbf{v}_t^* + \boldsymbol{\varphi}_t\left(\boldsymbol{\iota} U_t(1+c) - \mathbf{v}_t^*\right). \tag{5.11}$$

[9] The time steps in this section, as well as in the remainder of this chapter, are equal to one month. The described method, however, also allows other time steps.

The same argument applies for the mortgagee with one exception: whenever the mortgagor prepays the contract, the value U_t, is used instead of $U_t(1+c)$. The contract value for the mortgagee can consequently be calculated by:

$$\mathbf{w}_t^* = \mathbf{B}\left(\mathbf{w}_{t+1} + \iota M_0\right), \tag{5.12}$$

$$\mathbf{w}_t = \mathbf{w}_t^* + \boldsymbol{\varphi}_t\left(\iota U_t - \mathbf{w}_t^*\right). \tag{5.13}$$

This backward moving process will eventually bring us to the value at origin. The present value of the contract for the mortgagor and mortgagee is found by selecting the i^{th} element of $\mathbf{v}_0$ and $\mathbf{w}_0$, respectively, where i corresponds with the state of the economy at origination.

An important property of the prepayment boundary in Equation (5.10) is that it is entirely determined by the spot rate process, without any reference to refinance alternatives open to the borrower at time t. And because the term structure only depends on the spot rate in this one-factor economy , there is no role for the mortgage rate $y(t)$ at which the borrower can obtain a new mortgage loan for the remaining time to the reset date. In fact, the new mortgage rate $y(t)$ must be a function of the short-term interest rate $r(t)$, such that the new mortgage has a value equal to the principal amount.

5.3.2 Moneyness boundary

The suboptimal prepayment rule applied in this chapter assumes that the mortgage will be prepaid as soon as this reduces the total future costs for the borrower. This prepayment behavior is explained in detail in Section 5.2.4. There we also explained how the critical mortgage rate $y_c(t)$ can be determined for each initial mortgage rate $y(0)$. If the mortgage rate is below the critical rate for a particular month we assume that the mortgage will be prepaid and the value U_t is fed into the corresponding node of the interest rate tree. Hence, we do not have to make a distinction between the contract value for the mortgagor and mortgagee. Instead, we can directly form the diagonal prepayment matrix $\boldsymbol{\varphi}_t$ where the j^{th} element on the diagonal position is equal to:

$$\varphi_{jt} = \begin{cases} 1 & \text{if} \quad y_i(t) < y_c(t), \\ 0 & \text{if} \quad y_i(t) \geq y_c(t). \end{cases} \tag{5.14}$$

Subsequently, the mortgage value can be determined by using Equations (5.12) and (5.13).

5.4 Interest rate risk

The interest rate risk is measured by the *effective* duration Δ which is defined as the semi-elasticity of the mortgage value with respect to the spot interest rate:[10]

$$\Delta(r_j, y_j) = -\frac{v(r_{j+1}, y_j) - v(r_{j-1}, y_j)}{(r_{j+1} - r_{j-1})v(r_j, y_j)} \times 100. \tag{5.15}$$

We adjusted the notation for the mortgage value to emphasize that the initial short-term interest rate is adjusted while the mortgage rate remains untouched. In Equation (5.15), $v(r_{j+1}, y_j)$ represents the mortgage value at time $t = 0$ issued with a contract rate corresponding with state j, while the discount rate corresponds with state $j + 1$.

The effective duration is a better measure of the interest rate sensitivity of a callable contract than the duration.[11] However, the duration still gives us a good idea about the weighted time to maturity, which is important to know for the funding of a mortgage. Therefore we also report the duration measure in this chapter.

Duration analysis was first described by Macaulay (1938) using the present value of the cash flows as weights, such that:

$$D = \frac{\sum_{t=1}^{T} \frac{tC_t}{(1+r)^t}}{\sum_{t=1}^{T} \frac{C_t}{(1+r)^t}}, \tag{5.16}$$

where D is the duration, T is the time to maturity, C_t is the cash flow at time t and $(1+r)^t$ is the relevant discount factor. The denominator is simply the price of the mortgage, while the numerator is the present value of the cash flows weighted according to the time to cash receipt. This numerator can be determined simultaneously with the value of the contract. In Figure 5.1, a two-period example is worked out to illustrate this. There we assume that the mortgage will be prepaid at time $t = 1$ if state $i = 4$ occurs. The cash flow at time $t = 2$ is equal to M_0, such that the weighted values in the corresponding nodes become $2M_0$. If we move one period back and consider node $i = 2$ we know that the mortgage is not prepaid such that the cash flow is equal to M_0. And of course we have to take the weighted present value of future cash flows into consideration. In node $i = 4$ at time $t = 1$ the mortgage is called and the lender receives $(M_0 + U_1)$. Future cash flows are not relevant anymore and the corresponding value in this node becomes $1(M_0 + U_1)$. To derive the value of the numerator of Equation (5.16) at origination, we have to make one additional step backwards from both nodes at time $t = 1$ to node $i = 3$ at time $t = 0$. Since no payment is made at $t = 0$ we only have to discount the expected values of time $t = 1$.

In Appendix 5.A we present an alternative algorithm to derive the duration of a callable mortgage contract. That algorithm makes explicit use of the probability that the mortgage has or has not been called prior to the time period under consideration, such that also the "life expectations "of a mortgage contract can be determined.

[10] See Fabozzi and Modigliani, Chapter 13 (1992).

[11] The duration is inappropriate because the expected cash flows of a mortgage change as interest rates fluctuate. See DeRosa, Goodman and Zazzarino (1993) and Choi (1996) for a more detailed discussion on this topic..

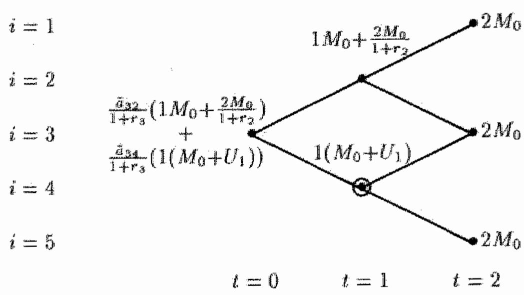


FIGURE 5.1: DURATION CALCULATION

5.5 Description of the data

The Dutch interest rates we use in this paper are end-of-month observations obtained from DataStream. The data are used to model the interest rate dynamics and the relation between short-term interest rates and mortgages rates. Data for the Dutch one-month Holland Interbank rate are available beginning January 1981. The five-year yield on government bonds and the Dutch mortgage rate are available from January 1975 and January 1974, respectively. The mortgage yield is the average yield on annuity-mortgages whose contract rates are fixed for a 5-year period. Figure 5.2 plots the different interest rates for the overlapping period; summary statistics are reported in Table 5.1.

TABLE 5.1: SUMMARY STATISTICS OF DUTCH INTEREST RATES

Series	Mean	Std. Dev.	Max.	Min.
Interbank (a)	7.10	2.07	13.81	3.81
5-year bond (b)	7.71	1.68	12.47	4.90
Mortgage Rate (c)	8.86	1.61	13.45	6.51
Spread (b-a)	0.61	1.28	4.00	-1.89
Spread (c-a)	1.76	1.19	4.71	-0.85
Spread (c-b)	1.15	0.33	1.90	0.32

Sample period: January 1981 - December 1994

The Dutch yield curve is relatively flat with an average spread of only 61 basis points. The mortgage rate commands a 115 basis point spread over the five year government bond yield. The five year government yield and the mortgage rate follow each other closely, except at times when the government yield rises sharply.

To estimate the empirical relation between the American mortgage rate and the short-

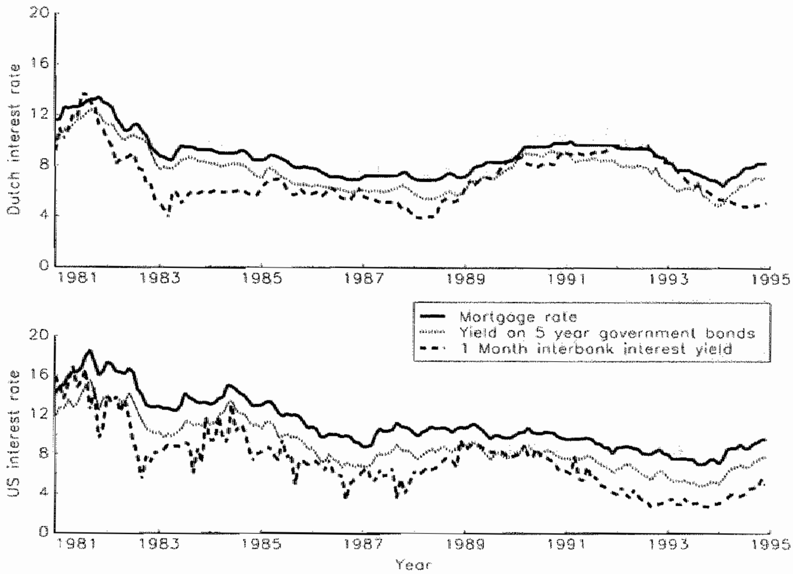


FIGURE 5.2: TIME SERIES OF DUTCH AND AMERICAN INTEREST RATES

The Dutch interest rates and US mortgage rates are end-of-month observations obtained from DataStream. For the other US data we have used bond prices available on CRSP tapes.

TABLE 5.2: SUMMARY STATISTICS OF AMERICAN INTEREST RATES

Series	Mean	Std. Dev.	Max.	Min.
1-month bond (a)	7.28	3.20	16.97	2.59
5-year bond (b)	8.88	2.50	15.50	4.80
Mortgage Rate (c)	11.11	2.67	18.55	7.02
Spread (b-a)	1.59	1.53	6.01	-4.20
Spread (c-a)	3.83	1.56	8.68	-1.66
Spread (c-b)	2.24	0.38	3.66	1.52

Sample period: January 1981 - December 1994

term interest rate we have used US bond data, available from CRSP tapes for the period January 1970 until December 1994. The data, which are summarized in Table 5.2, were collected on a monthly basis. The US mortgage rate is obtained from DataStream. This mortgage rate is an average of the secondary market yields on FHA (= Federal Housing Associations) mortgages. It is an average of all yields to maturity for different mortgages.

Comparing both tables we see that the American interest rates are more volatile than their Dutch counterparts. Additionally, the historical spread between the mortgage rate and the 5 year yield is almost 95% larger in the US than in the Netherlands. And also the spread between the mortgage rate and the short-term interest rate is much larger in the US than in the Netherlands. This latter is illustrated in Figure 5.3.

5.6 Spot rate dynamics

Here we consider three specifications for the dynamics of the spot rate: the now familiar CIR square root process, a flexible nonlinear time series model and a nonparametric model. All three processes will be used in the mortgage valuation to study the sensitivity of the results for the underlying interest rate model.

5.6.1 The CIR model

The starting point for modelling interest rate derivative securities is usually a diffusion process of the form:

$$dr = \mu(r,t)dt + \sigma(r,t)dz, \tag{5.17}$$

where the functions $\mu(\cdot)$ and $\sigma(\cdot)$ are the instantaneous proportional drift and the volatility of the interest rate, respectively, and dz is a Wiener process. The CIR model is the most widely-applied term structure model. It is a mean-reverting model where the speed-of-adjustment to the unconditional mean, θ, in an interval dt is measured by κ. The variance

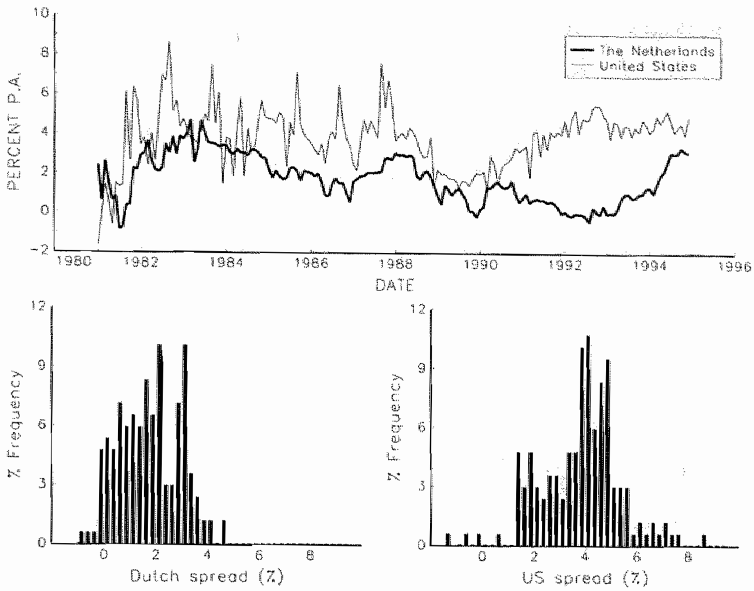


FIGURE 5.3: MORTGAGE SPREAD

The upper diagram of this figure shows the historical spread between the mortgage rate and the short-term interest rate in the Netherlands and the United States, respectively. The lower diagrams show the frequency distributions of these spreads.

in this model is proportional to the level of the instantaneous risk-free rate:

$$dr = \kappa(\theta - r(t))dt + \sigma\sqrt{r(t)}dz. \tag{5.18}$$

The empirical problem is to estimate these parameters for the CIR interest model. The usual procedure to do this is by time series analysis. Chan, Karolyi, Longstaff and Sanders (1992) give an empirical overview of the results of time series methods. Here the parameters are estimated by Maximum Likelihood using the time series data for the one-month Interbank rate between January 1981 and December 1994.[12] Table 5.3 contains the estimated parameters for the CIR model.

The conversion of the continuous time CIR model into a discrete, finite Markov chain has been introduced by Hull and White (1990) and is summarized in Appendix 5.B.

The risk-neutral parameters depend on the assumption on the market price of risk λ, which can be identified through the infinite horizon yield. This constant is independent of the current spot rate:

[12] The CIR process is based on the instantaneous risk-free rate which cannot be observed. For the closest approximation the one-month interest rate is used. A discrete time representation of the spot rate process is utilized to estimate the CIR parameters for the the time series process of the one-month interest rate; see De Munnik and Schotman (1994).

TABLE 5.3: CIR PARAMETERS

	κ	θ	σ
Estimate	0.38	6.33	0.49
$\lvert t\rvert$-statistic	1.77	4.34	3.62

Sample: Jan. 1981 - Dec. 1994

$$R_\infty = \frac{2\kappa\theta}{\tilde{\kappa} + (\tilde{\kappa}^2 + 2\sigma^2)^{\frac{1}{2}}}, \tag{5.19}$$

where $\tilde{\kappa} = \kappa + \lambda$ (see Cox, Ingersoll and Ross, 1985b). We compute λ and thus $\tilde{\kappa}$ by setting $R_\infty = 8\%$.

5.6.2 A nonlinear model

A more general specification of the interest rate process is the parametric one-factor model based on Aït-Sahalia (1996a,b). The Euler discretization of the continuous time model developed by Aït-Sahalia reads:

$$r(t+1) = \alpha_0 + \alpha_1 r(t) + \alpha_2 r(t)^2 + \alpha_3 \frac{1}{r(t)} + \sqrt{s_0 + s_1 r(t)}\epsilon_{t+1}. \tag{5.20}$$

The model allows for heteroskedasticity of the constant elasticity of variance (CEV) type, and is mean-reverting if either $\alpha_2 = 0$ and $\alpha_1 < 0$ or $\alpha_2 \leq 0$. In the latter case the model is strongly mean-reverting at high interest rate levels. With $\alpha_3 > 0$ the interest rate has a strong upward drift close to zero, thus excluding negative interest rates. Maximum likelihood parameter estimates for the Dutch time series are presented in Table 5.4.

TABLE 5.4: NONLINEAR PARAMETERS

	α_0	α_1	α_2	α_3	s_0	s_1
Estimate	-8.693	1.120	-0.047	21.42	0.109	0.00191
$\lvert t\rvert$-statistic	2.76	4.76	2.35	3.08	4.06	3.91

Sample period: January 1981 - December 1994

The parameter estimates are not very informative about the implications of this process. Since the model has nonlinear dynamics, its properties can not be established by analytical methods. Numerical methods for analyzing this model include a long simulation and discretization of the probability space.

The latter method involves dividing the range of possible values of the interest rate into a (large) number of intervals. Starting from each interval we can compute the probability of the interest rate moving to any other interval. The result is a discrete time Markov chain, which lends itself to the analysis of the dynamics of the interest rate as well as to the valuation of a mortgage. Pfann, Schotman and Tschernig (1996) apply this method to a nonlinear model of the US term structure, while Benninga and Protopapadakis (1994) use it for pricing interest rate futures. Taking the number of states in the CIR model into account, we fixed the number of states for the nonlinear model at $N = 226$.

The nonlinear interest rate model implies a mean interest rate of 6.3%, and a volatility of 1.9%. (Note that this is before the market price of risk is included.) The drift of the model is plotted in Figure 5.4.

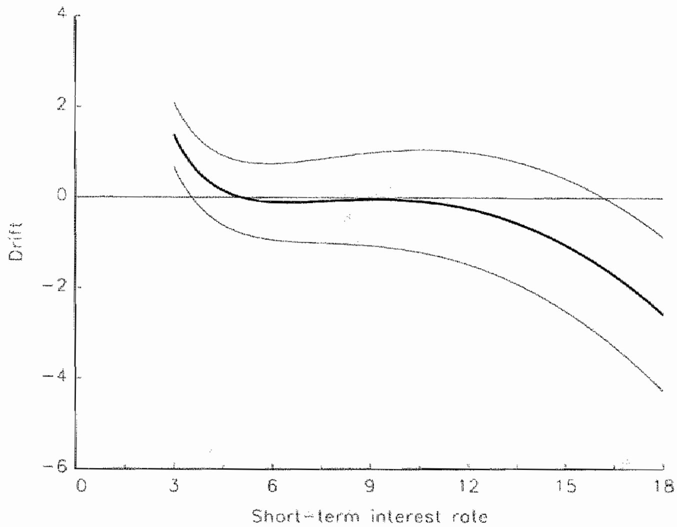


FIGURE 5.4: THE NONLINEAR MEAN-REVERTING DRIFT

The bold line in this figure shows the drift resulting from the nonlinear interest rate model estimated on Dutch data. The thin lines illustrate the 95% confidence interval.

Figure 5.4 shows that there is almost no mean-reversion when the spot rate is in the range of 5 to 10%. In this region the spot rate behaves mainly like a random walk. When the spot rate is outside this region the mean-reverting drift gets strong and pulls the rate back into the 5 to 10% range. These findings are consistent with the results found by Aït-Sahalia (1996a,b) for the American spot rate. Unlike Aït-Sahalia's American data, where the highest spot rate was about 24%, our Dutch spot rate time series does not contain rates higher than 13.81%. Our nonlinear model should therefore be carefully approached at high interest rates.

As discussed in Section 5.3, to include the market price of risk we adjust the drift such that the implied infinite horizon yield R_∞ equals 8%. This results in an interest rate model which has a bimodal distribution as can be seen in the bottom right quarter of Figure 5.5.

The top half of this figure displays the speed at which different interest rates converge

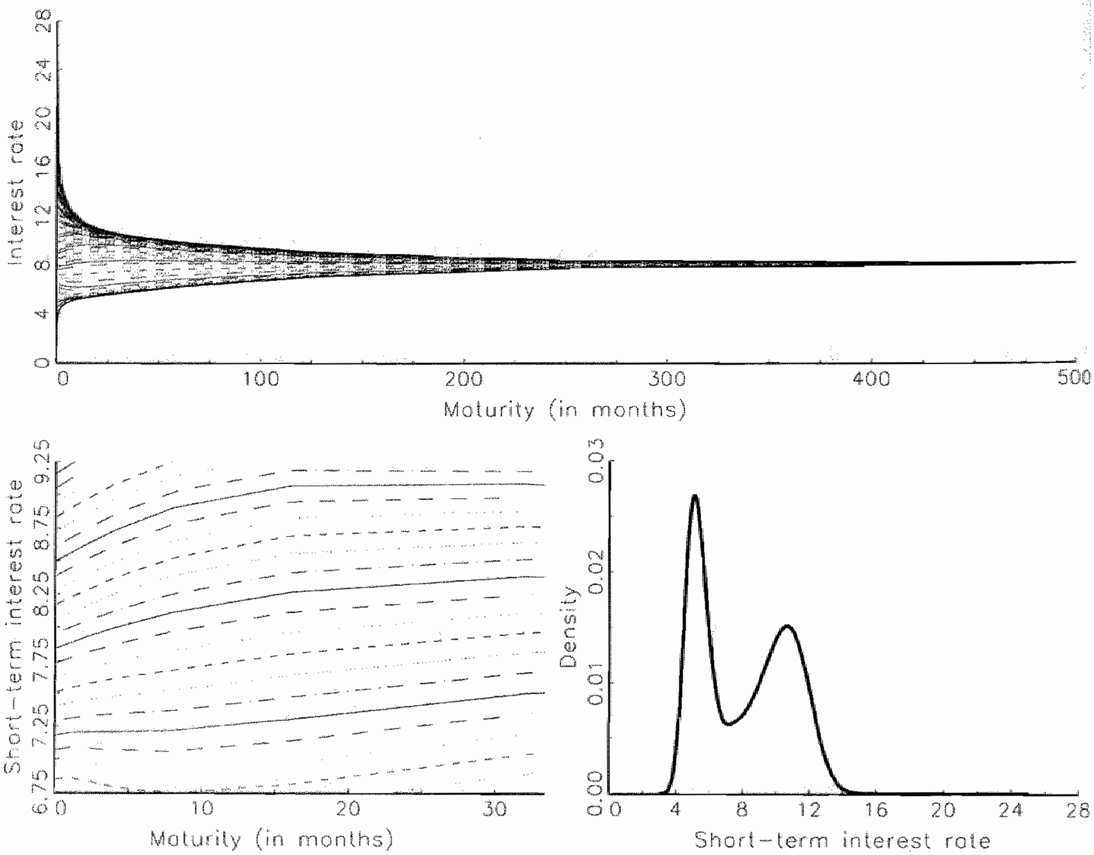


FIGURE 5.5: CHARACTERISTICS OF THE NONLINEAR INTEREST RATE MODEL

The upper diagram of this figure shows the term structure of interest rates for different starting rates. The bottom left quarter yields a close up look of the top figure. The bottom right plots the risk-neutral unconditional distribution.

to the long-term mean. We see that extreme interest rates are firmly pulled back into the middle range after which the speed of mean-reversion decreases. In the bottom left quarter of Figure 5.5 we take a close up look of the top figure. This enlargement summarizes both other figures. A trend towards a short-term interest rate of 8 percent is observable. At the same time we see that small deviations in the starting rate make a large difference in the path the process is expected to follow afterwards. Starting at a spot rate of 7.5 percent the process is expected to follow a concave path towards the long-term mean, while starting at 7 percent the expected path has a convex beginning. The difference between a concave and a convex interest path is of major importance for the prepayment likelihood so that even small differences in the starting spot rate can yield serious deviations in the mortgage values.

5.6.3 A nonparametric density estimation

For the nonparametric approach, no assumptions need to be made regarding the functional form of the probability density function (except for some regularity conditions on smoothness of the density). In principle, model misspecification is excluded, however, estimation error is unavoidable.

Following the approach of Aït-Sahalia (1996a,b) a kernel method is used to obtain the conditional transition density of the spot rate.[13] One can compare kernel estimation with smoothing a histogram. The density at a point is estimated as the average of densities centered around that point. Observations farther away from the estimation point still contribute to the estimated density but to a lesser degree than observations closer to this estimation point. As a result, the density will be highest near concentrations of observations, while the density will be low when observations are scarce.

The nonparametric kernel estimator of the marginal density reads :

$$\hat{\pi}(u) \equiv \frac{1}{T}\sum_{t=1}^{T}\frac{1}{h_T}K\left(\frac{u-r(t)}{h_T}\right), \tag{5.21}$$

where h_T is the bandwidth, also called the window width or smoothing parameter. The basic idea behind kernel smoothing is to find a probability density function for $K(\cdot)$ that describes the diffusion process well. Here, we choose a Gaussian kernel $K(u) = \frac{1}{\sqrt{2\pi}}\exp(-\frac{1}{2}u^2)$ to smooth the densities. The quality of the density estimator is highly dependent on the choice of the bandwidth. Following Härdle (1993), the bandwidth is defined as $h_T = d_T T^{\frac{-1}{5}}$ where T is the number of observations in our time series, and d_T equals d times the standard deviation of the spot rate time series. The smoothing parameter d is chosen by cross-validation to minimize the integrated squared error of the estimator. The smoothing parameter regulates the size of the neighborhood around the dependent variable. When

[13] Härdle (1990), Scott (1992) and Silverman (1986) provide an extensive explanation of kernel estimation.

this neighborhood is too large, an oversmoothed transition matrix is constructed, while too small a neighborhood will result in undersmoothing.

Bayes' rule tells us that

$$\hat{p}(w \mid v) \equiv \frac{\hat{p}(w, v)}{\hat{\pi}(v)}, \tag{5.22}$$

where v and w are class midpoints. The joint density of observations is written as $p(w, v)$. In the above equation this is replaced by its kernel estimator $\hat{p}(w, v)$. Similarly, $\hat{p}(w \mid v)$ is the kernel estimator of the conditional transition probability. As shown by Aït-Sahalia (1996a), Equation 5.22 can be replaced by its kernel estimator:

$$\hat{p}(w \mid v) = \frac{\frac{1}{T}\sum_{t=1}^{T} \frac{1}{h_T^2} K\left(\frac{v-r(t)}{h_T}\right) K\left(\frac{w-r(t+1)}{h_T}\right)}{\frac{1}{T}\sum_{t=1}^{T} \frac{1}{h_T} K\left(\frac{v-r(t)}{h_T}\right)}. \tag{5.23}$$

Figure 5.6 shows the conditional density as it results from the nonparametric approach. Between January 1981 and December 1994 the one-month interest rate did not rise above 14 percent. In a nonparametric setting this results in a strong mean-reversion force at high interest rates, as becomes obvious in Figure 5.7.

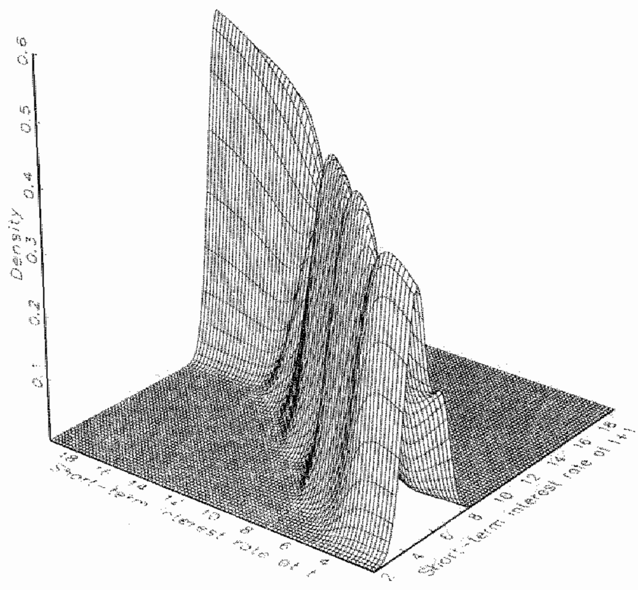


FIGURE 5.6: NONPARAMETRIC TRANSITION DENSITIES

The upper diagram of Figure 5.7 displays the contours of the conditional density; the lower diagram plots the drift. This lower diagram clearly shows that high interest rates are firmly pulled back into a range between 6 and 12%, while in this range the mean-reversion is negligible. A similar result was found for the nonlinear model.

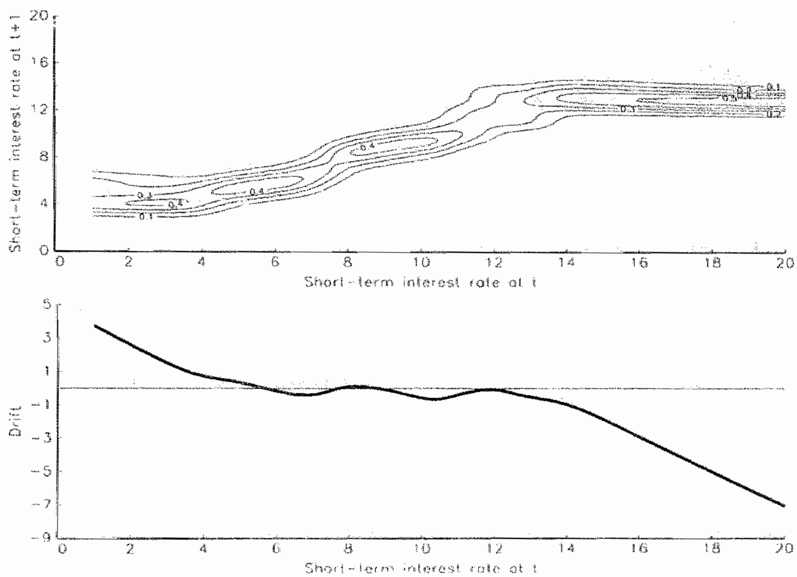


FIGURE 5.7: CHARACTERISTICS OF THE NONPARAMETRIC INTEREST MODEL

The upper diagram of this figure shows the contours of the nonparametric transition densities. In the lower diagram the nonparametric drift is plotted.

5.7 Mortgage rate dynamics

The importance of the mortgage rate dynamics was already stressed in Section 5.2.3. The relation between the mortgage rate and short-term interest rate is endogenously derived for both the backward and forward valuation method. In the forward-looking valuation approach we also consider two empirical relations between the spot rate and the mortgage rate.

We start with an estimation of the empirical relation between the mortgage rate and the short-term interest rate in the Netherlands. The idea is to fit a function

$$y(t) = f(r(t)), \tag{5.24}$$

where $y(t)$ is the mortgage rate and $r(t)$ the short-term interest rate at time t. The function $f(r(t))$ can be highly nonlinear and is therefore estimated nonparametrically. The left diagram of Figure 5.8 shows the result for the Netherlands. The thick line in the figure will be used as the empirical equilibrium relation between the mortgage rate and the spot interest rate.

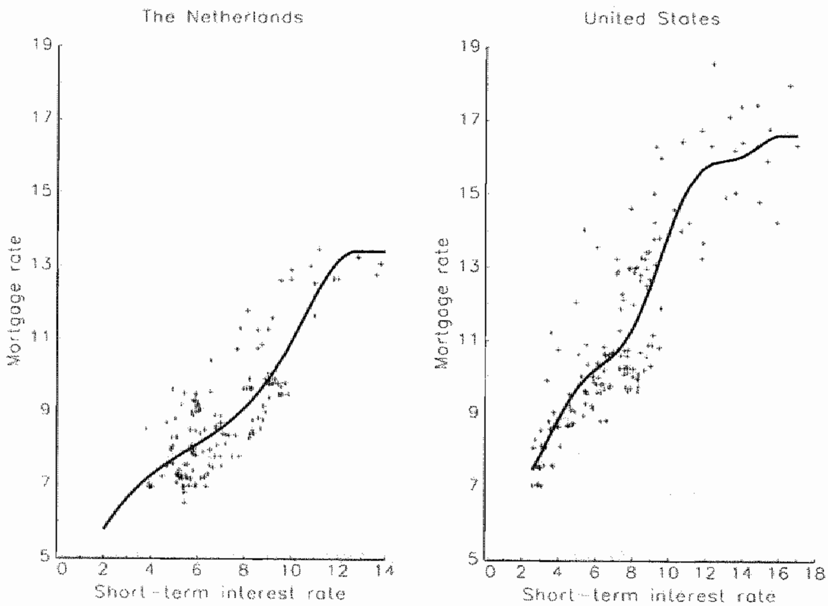


FIGURE 5.8: RELATION BETWEEN THE MORTGAGE RATE AND THE SPOT RATE

This figure shows the relation between short-term interest rates and mortgage rates for both the Netherlands and the US. The "crosses" depict a scatter of the observations. The solid lines are the estimated functional relations.

The Dutch curve shows that the mortgage premium over the short-term interest rate is more than 300 basis points at low levels of r, and drops to about 110 basis points at interest rates above 10%. The premium consists among others of a compensation for the prepayment risk embedded in Dutch mortgages. However, these mortgages have very rigid prepayment restrictions which are undoubtedly reflected in the historical spread. In this chapter we analyze what would happen if these restrictions would be abandoned. The historical premium might therefore be too small to cover the risks involved in the contracts studied. American mortgages have less restrictions on prepayment so that the historical American relation between the short-term interest rate and mortgage rate might be more appropriate here. However, this relation reflects the US interest rate dynamics which might substantially differ from the Dutch dynamics.

In keeping with Tables 5.1 and 5.2, the diagrams in Figure 5.8 illustrate that American interest rates attained higher levels than their Dutch counterparts in the considered period. Consistent with the less restricted prepayment options, the US mortgage spread is higher than the Dutch spread.

Both the Dutch and American figure will be used to describe the mortgage rate dynamics underlying our valuation model. Furthermore, we will endogenously determine the contract rate which makes a mortgage a derivative security with a zero net present value. This is essentially the approach taken by Kau, Keenan, Muller and Epperson (1993).

5.8 Optimal prepayment results

This section presents the results of the valuation procedure based on the optimal prepayment rule. We start with the derivation of the mortgage rate which equals the mortgage value at origination to the principal of the loan. Recall that in this chapter we study a mortgage which can be completely called. However, prepayment is discouraged by the up-front costs for starting a new loan necessary to refinance the existing contract. These up-front costs are assumed to be 1% of the outstanding balance.

The endogenous relationship between the mortgage rate and the short-term interest rate is determined with the CIR, the nonlinear and the nonparametric models describing the underlying short-term interest rate dynamics. The results are shown in Figure 5.9. This figure shows that the CIR model results in an endogenous relation between the mortgage rate and the short-term interest rate which is close to linear, while for both alternative models a nonlinear relation comes out. The impact of the alternative single factor models is most obvious at low interest rates. The differences are caused by the different drift terms in the models. For example, Figures 5.4 and 5.7 show that low spot rates are firmly pulled back into a range of between 6 and 10 percent by both the nonlinear interest rate model and the nonparametric model. The exact size of this upward drift depends on the underlying model. It is therefore not surprising that the various endogenous relations converge at rates below 6%. The upward drift is strongest in the nonparametric model and therefore this model implies the highest mortgage rates at low short-term interest rates.

The crosses in the left diagram of Figure 5.9 depict a scatter of observations in the

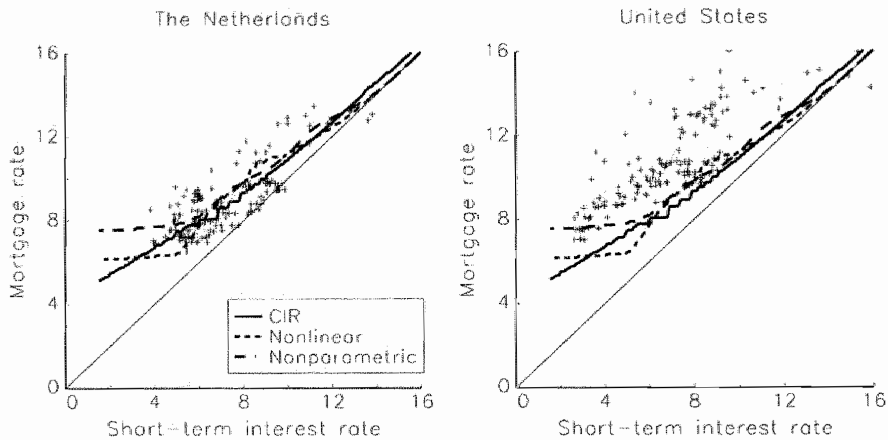


FIGURE 5.9: ENDOGENOUS MORTGAGE RATES

The mortgage rates plotted in this figure are endogenously derived such that the value of the mortgage contract equals the face value of the loan. In this setting a mortgage is an investment with an expected net present value of zero. The optimal prepayment rule is used to model the prepayment behavior. The endogenous relations are shown for alternative specifications of the underlying interest rate dynamics. The crosses in the left diagram depict a scatter of observations in the Netherlands, in the right diagram they represent US observations. The endogenous relations presented in both diagrams are based on Dutch interest rate dynamics.

Netherlands. Note that these observed mortgage rates refer to contracts with annual prepayment restrictions. The endogenous relation, on the other hand, is based on the assumption that the mortgage can be completely called each month. One would therefore expect that the observed mortgage rates would all be below the various endogenous relations. But Figure 5.9 shows that the observations are scattered around the endogenously derived relations which suggest that a typical Dutch mortgage contract is not a zero net present value investment.

American mortgages have less prepayment restrictions and therefore resemble the contract studied here. In the right diagram of Figure 5.9 we compare the endogenous relations with US observations. This time we expect the crosses to be scattered around the various endogenous relations, but Figure 5.9 shows that almost all US mortgage rates observed between January 1981 and December 1994 exceed the endogenously derived rates. Note that the endogenous relations are based on Dutch interest rate dynamics while the US observations are of course based on American dynamics. As Table 5.2 in Section 5.5 shows, the volatility of the American interest rates is relatively high compared with their Dutch counterparts. Apparently this has a substantial impact on the spread between mortgage rates and short-term interest rates, such that we have to be reserved to use American interest rate relations to value Dutch contracts.

In Figure 5.10 we used the endogenously derived mortgage rates to determine the refinancing boundaries. These boundaries depend on the interest rate process underlying the backward valuation algorithm. Hence, Figure 5.10 displays the critical boundaries for the CIR, nonlinear and nonparametric model. The results are shown for initial short-term interest rates of 4, 8 and 12 percent and for five levels of refinancing costs.

In keeping with economic theory, Figure 5.10 shows that the critical boundary increases as the refinancing cost decreases. Similar to McConnell and Singh (1994) we find that the critical boundary increases through time when there are no transaction costs:

> *...for the zero refinancing cost category, the critical boundary traces the level of the short-term interest rate that gives the current coupon mortgage rate for a mortgage with a term to maturity equal to the remaining term of the original mortgage. As the remaining term to maturity declines, the current coupon rate approaches the short-term rate. At maturity, the critical refinancing rate equals the original coupon rate.*

For positive transaction costs we see that eventually the critical rate declines to zero. This means that, independent of the prevailing interest rate, the remaining time to the reset date is too short for the savings on the monthly payments to make up for the transaction costs.

Table 5.5 examines a mortgage contract for which the only discouragement to prepay arises from one percent refinancing costs. The table presents the results for a noncallable annuity and a mortgage contract. Given the underlying interest rate process, the mortgage rate is endogenously derived such that the value of the mortgage contract at origination

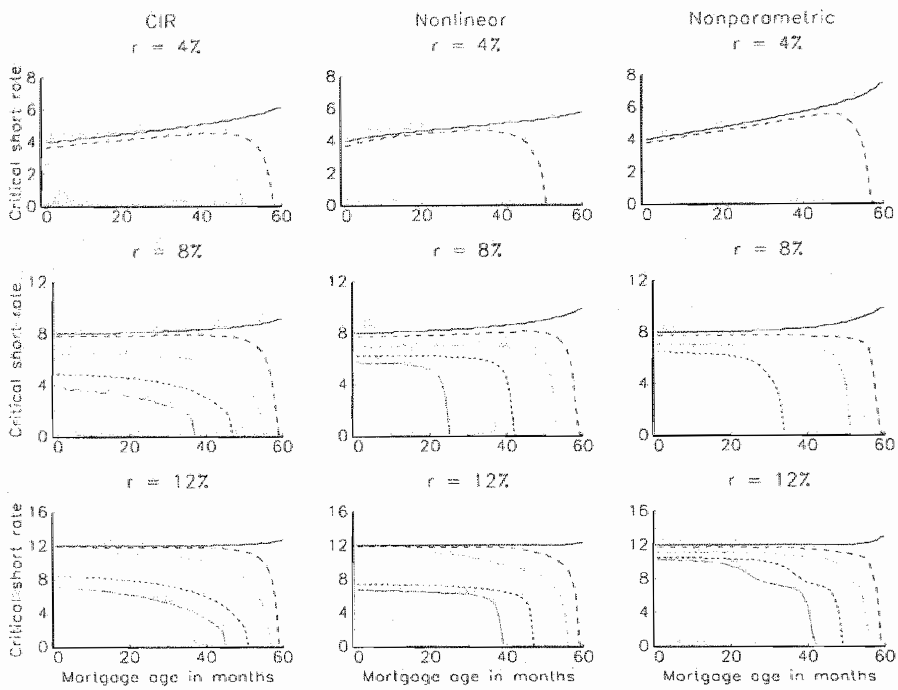


FIGURE 5.10: CRITICAL SHORT-TERM INTEREST RATES

This figure plots the critical short-term interest rates as derived under the optimal prepayment rule. The results are based on the endogenously derived mortgage rates corresponding with short-term interest rates of 4, 8, and 12 percent, respectively. The diagrams in the first column are based on the CIR model, while the second and third column are based on the nonlinear and nonparametric interest rate process, respectively. The refinancing costs as a percentage of the outstanding balance are, from the highest to the lowest boundary, 0, 1, 3, 6 and 9 percent, respectively.

TABLE 5.5: OPTIMAL PREPAYMENT RESULTS

Spot Rate	4%	8%	12%
		CIR model	
Annuity value	103.56	104.96	108.53
Duration annuity	50.04	48.09	45.83
Duration mortgage	30.15	8.82	3.54
Δ annuity	2.18	2.06	1.94
Δ mortgage	0.05	0.18	0.20
		Nonlinear model	
Annuity value	102.80	105.71	106.99
Duration annuity	50.41	47.76	45.86
Duration mortgage	23.23	13.92	2.48
Δ annuity	0.26	3.94	0.73
Δ mortgage	0.09	0.50	0.12
		Nonparametric model	
Annuity value	102.12	106.82	113.91
Duration annuity	49.11	47.85	46.11
Duration mortgage	19.91	8.90	3.70
Δ annuity	0.24	1.42	0.85
Δ mortgage	0.13	0.24	0.11

The table shows the valuation results for an annuity and a mortgage contract. Given the interest rate dynamics implied in the underlying interest rate model, the mortgage rate is endogenously derived such that the value of the mortgage contract at origination equals the face value of the loan. Hence, the mortgage value is 100 by design. Duration, expressed in months, and Δ are defined in Section 5.4. Both measure the interest rate sensitivity of the mortgage contract.

equals the face value of the loan. Hence, the mortgage value is 100 by design and not presented in the table. The value of the prepayment option is also not shown because it can be found by simply subtracting the value of the mortgage from the value of the annuity contract.

Table 5.5 shows that the different underlying interest rate processes yield very similar valuation results. Even though the exact figures differ slightly, similar patterns may be recognized. For example, the value of the annuity and prepayment option increase along with the short-term interest rate, while the duration decreases with increasing interest rates. The Δ measures show varying patterns depending on the model used. The linear drift in the CIR model keeps the Δ measures more or less the same for different initial short-term interest rates. The nonlinear and nonparametric models, on the other hand, have a nonlinear drift which pulls high and low interest rates firmly back into a range of between 6 and 10 percent. Within this range, the interest rates behave like a random walk and this uncertainty is reflected by the relatively high interest rate sensitivity of the mortgage contract at the 8 percent interest rate level.

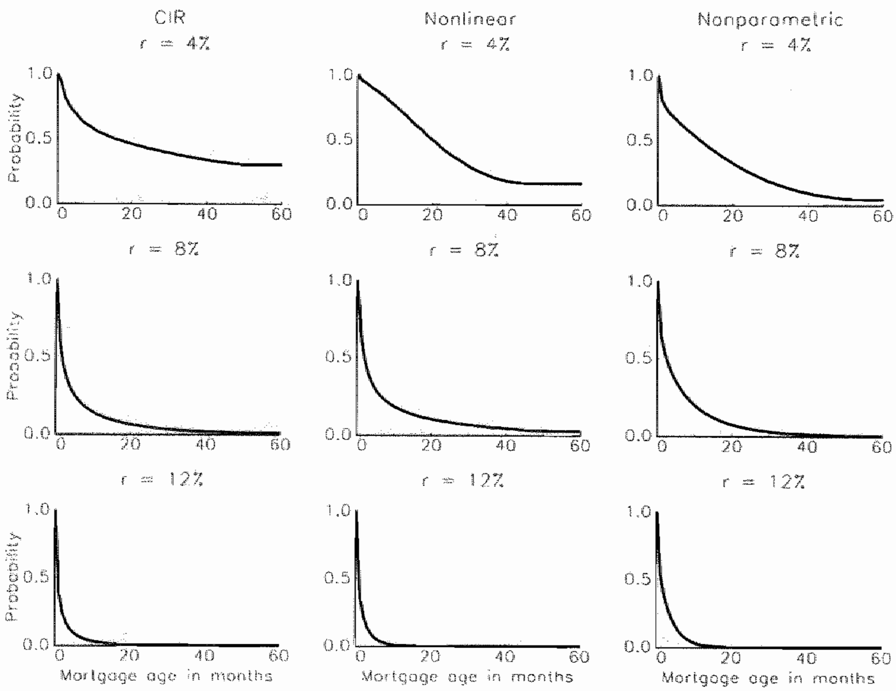


FIGURE 5.11: PROBABILITY OF NOT BEING PREPAID

These figures show the probability that the mortgage has not been prepaid at a specific age of the contract. The refinancing costs are assumed to equal 1 percent of the outstanding balance.

The interest rate risk faced by the lender is closely linked to the uncertainty as to when the money will be received: Will the borrower follow the periodic principal repayments scheduled over the entire time to maturity, or will the mortgage be called prematurely? In Figure 5.11 we study this by looking at the probability that a mortgage has not been prepaid at a specific age of the contract. The results are shown for the three alternative underlying interest rate models at spot rates of 4, 8 and 12 percent, respectively. Even though the "life expectations" of a mortgage contract depend on the underlying stochastic interest rate process, Figure 5.11 shows that at moderate and high interest rates it is almost certain that a mortgage will be prepaid before the fixed-rate period of 5 years ends. Also at lower interest rates the various single factor models agree that there is a significant probability that the mortgage is prepaid before the reset date.

5.9 Suboptimal prepayment results

This section presents the valuation results based on the suboptimal rule where prepayment is triggered as soon as refinancing reduces the future costs for the borrower. The results are presented for the three alternative one-factor interest rate models. The one-factor assumption implies a one-to-one correspondence between the spot rate and the mortgage rate. We will discuss three specifications for this functional relation. Two exogenous relations will be considered, while for the third specification the mortgage rate is endogenously determined.

5.9.1 Exogenous interest rate relations

Table 5.6 shows the computed value at origination of the mortgage contract, the value of the prepayment option, and two measures of interest rate risk: duration and Δ. For straight bonds, duration and Δ both measure the sensitivity to interest rate changes and are readily interchangeable. For a mortgage, the two concepts have very different properties. For comparison purposes, the results of a noncallable annuity are also included. The results are presented for the 4, 8 and 12 percent level of the spot rate. The matching contract rates are based on both the Dutch and American empirical relation between the short-term interest rate and mortgage rate, as described in Section 5.7.

The results presented in the upper panel of Table 5.6 are calculated with the Cox, Ingersoll and Ross model describing the underlying short-term interest rate dynamics. Because of the mean-reversion implied in the CIR model, the noncallable annuity is expected to increase with interest rates. However, when the historical Dutch relation between the short-term interest rate and the mortgage rate is used, Table 5.6 shows that the annuity value is higher when the short-term interest rate is 4 percent than when it equals 8 percent. This is caused by the different margins between the short-term interest rate and the mortgage rate. As illustrated in Figure 5.8, the margin rapidly decreases from 300 to 110 basis points when the short-term interest rate increases from 4 to 8 percent, while for rates above 8 percent the margin stays around 110 basis points.

TABLE 5.6: MORTGAGE VALUES AND THE INTEREST RATE PROCESS

Empirical Spread	Dutch			American		
Spot Rate	4%	8%	12%	4%	8%	12%
			CIR model			
Annuity value	105.75	104.07	110.12	111.19	113.77	116.78
Mortgage value	104.38	100.29	100.26	109.44	102.78	101.25
Option value	1.38	3.78	9.86	1.75	10.99	15.53
Duration annuity	49.72	48.22	45.61	48.95	46.84	44.72
Duration mortgage	41.16	16.38	5.85	43.10	13.62	6.66
Δ annuity	2.17	2.07	1.94	2.14	2.02	1.91
Δ mortgage	-1.01	0.14	-0.09	-1.88	-3.06	-1.14
			Nonlinear model			
Annuity value	106.78	103.08	109.40	112.24	112.72	116.05
Mortgage value	106.56	99.20	100.23	112.04	102.51	100.85
Option value	0.22	3.88	9.18	0.19	10.21	15.20
Duration annuity	49.84	48.15	45.52	49.07	46.76	44.62
Duration mortgage	48.35	19.01	3.75	48.40	16.91	4.20
Δ annuity	0.26	3.97	0.73	0.26	3.86	0.72
Δ mortgage	0.02	1.42	-0.03	0.09	-2.37	-0.45
			Nonparametric model			
Annuity value	100.25	104.17	114.77	105.48	113.86	121.60
Mortgage value	100.06	99.47	100.08	109.24	101.47	100.69
Option value	0.19	4.69	14.69	1.24	12.39	20.91
Duration annuity	49.40	48.24	46.00	48.62	46.85	45.12
Duration mortgage	36.90	11.23	4.08	38.91	10.20	4.33
Δ annuity	0.24	1.42	0.85	0.24	1.41	0.84
Δ mortgage	0.21	0.57	0.08	-0.15	-0.68	-0.13

The table shows valuation results for the three alternative short-term interest rate processes described in Section 5.6. For the spread between the short-term interest rate and the mortgage rate we used both the Dutch and American historical relation between these two rates. Duration, expressed in months, and Δ are defined in Section 5.4. Both measure the interest rate sensitivity of the mortgage contract.

At the 4 percent level the probability that the mortgage will be prepaid is small. Consequently, the mortgage contract studied here is similar to the contracts issued historically in the Netherlands. The historical Dutch premium might therefore be appropriate at this low interest rate. However, as the initial interest rate increases, the prepayment probability increases. The historical spread, based on restrictive contracts, might not be large enough to cover this prepayment risk. In that light it is surprising that, even if the relation between the spot rate and the mortgage rate is based on Dutch historical data, the model based on the CIR process still yields mortgage values above par for both the 8 and 12 percent spot rate.[14]

Contrary to the CIR model, the nonlinear and nonparametric interest rate models yield mortgage values below par. This would mean that the issuing party is accepting a loss. A more reasonable interpretation is that the Dutch historical premium is not appropriate. Therefore we also consider the empirical relation between the mortgage rate and the short-term interest rate in the US, where annual prepayment restrictions are virtually unknown. At moderate and high interest rates, the US relation shows a higher option premium which, as Table 5.6 shows, results in higher values for the annuity, the mortgage and the prepayment option. Furthermore, the mortgages all value above par.

Does this mean that the consequences of loosening the prepayment restrictions in Dutch mortgage contracts can correctly be analyzed by applying the empirical US relation? No. The negative Δ's reported in Table 5.6 reveal that the American empirical relation between the mortgage rate and the short-term interest rate is not appropriate for this purpose. A negative Δ measure indicates that the contract value *de*creases with decreasing interest rates!

Decreasing short-term interest rates result in decreasing discount rates. At the same time, the prepayment probability increases. Due to these callable features, the value of a mortgage contract will not increase as much as the value of a noncallable annuity, for example. One would expect that the value of a mortgage increases with decreasing interest rates until a value equal to the par value plus transaction costs is reached. Additional interest rate decreases would have no effect anymore. However, this is only the case when low initial short-term interest rates (and therefore low prepayment probabilities) are considered or when the mortgage value at origination is below or close to par. Otherwise the mortgage value decreases with decreasing interest rates. This is illustrated in Table 5.6 where mortgage values above par and moderate and high interest rates go hand in hand with negative Δ measures. These negative Δ's clearly reveal the shortcomings of exogenous approaches to relate the mortgage rate to the short-term interest rate.

[14] Instead of only implementing our point estimators for the mean-reverting and volatility parameter in the CIR model, we also studied the sensitivity of the mortgage value and its prepayment option for fluctuations in κ and σ. The volatility σ has a positive relation with prepayment probabilities. Hence, an increase in σ results in an increase in the option value. However, this is only noticeable at a short-term interest rate of 8%. At interest rates of 4 and 12%, the speed-of-adjustment factor κ dominates the volatility factor. Independent of these adjustments, the general results are very similar to the ones presented in the top panel of Table 5.6.

5.9.2 Endogenous interest rate relations

The endogenous relation between the mortgage rate and the short-term interest rate derived in Section 5.8 is based on the optimal prepayment rule. These mortgage rates are used in this subsection to value the mortgage contract in a framework where prepayment is triggered when this reduces the future costs for the borrower. Later we also derive the mortgage rates endogenously in this framework.

TABLE 5.7: VALUATION RESULTS

Spot Rate	4%	8%	12%
	CIR model		
Annuity value	103.56	104.96	108.53
Mortage value	102.38	100.23	100.06
Duration annuity	50.04	48.09	45.83
Duration mortgage	38.82	9.46	5.63
Δ annuity	2.18	2.06	1.94
Δ mortgage	-0.63	-0.02	0.16
	Nonlinear model		
Annuity value	102.80	105.71	106.99
Mortgage value	102.80	100.12	100.06
Duration annuity	50.41	47.76	45.86
Duration mortgage	50.41	16.79	4.64
Δ annuity	0.26	3.94	0.73
Δ mortgage	0.26	0.40	0.11
	Nonparametric model		
Annuity value	102.12	106.82	113.91
Mortage value	102.12	100.07	100.02
Duration annuity	49.11	47.85	46.11
Duration mortgage	49.11	11.66	4.67
Δ annuity	0.24	1.42	0.85
Δ mortgage	0.24	0.21	0.10

The table shows the valuation results for an annuity and a mortgage contract whereby it is assumed that the mortgage is called when refinancing reduces the future costs. The endogenous relation as derived under the optimal prepayment rule is used to relate the mortgage rates to the short-term interest rates.

Table 5.7 presents the valuation results of a mortgage contract which is assumed to be called as soon as this reduces the future costs for the borrower. The relationship between the mortgage rate and the short-term interest rate is endogenously derived under the optimal prepayment rule as described in Section 5.8. The most eye-catching results in Table 5.7 are the relatively high durations compared to their counterparts in Table 5.5.

This results in the counter-intuitive outcome that the prepayment likelihood is lower when the in-the-money rule is applied rather than the optimal prepayment rule! The explanation for this can be found in Figure 5.9, where the endogenously derived mortgage rates are plotted as a function of the short-term interest rate. Figure 5.9 shows that, regardless of the underlying interest rate process, the regression coefficients of the tangent lines at short-term interest rates of 4, 8 and 12 percent are smaller than the regression coefficient of a 45° line. This indicates that a one basis point decrease of the short-term interest rate corresponds with a decrease of the mortgage rate which is smaller than one basis point. This characteristic has a major impact on the prepayment behavior modelled by both approaches.

Recall that the prepayment boundary under the optimal prepayment rule is entirely determined by the spot rate, while the suboptimal prepayment boundary solely depends on the mortgage rate at which the borrower can refinance the loan. This distinction influences the prepayment behavior substantially. To see this let us consider a situation where the short-term interest rate drops with 100 basis points from 4 to 3 percent. The probability that this drop in the spot rate triggers prepayment when the optimal rule is used is substantial. The likelihood that the suboptimal rule prescribes prepayment is, however, much smaller. For example, when the nonlinear or nonparametric model is used, the adjustment in the mortgage rate is almost negligible, such that the probability that the mortgage is called is very small. This is also reflected by the higher duration measures. The same argumentation holds for short-term interest rate levels of 8 and 12 percent.

The prepayment boundary under the optimal prepayment rule does not depend on the refinance opportunities open to the individual. Hence, the mortgage rate corresponding to a particular short-term interest rate can be derived without reference to mortgage rates matching the other short-term interest rates. This is more complicated when the suboptimal prepayment rule is used. For example, suppose that the short-term interest rate at origination equals r_i, which is the short-term interest rate corresponding with state i. From this state i the short-term interest rate process can reach each state j in the next period. The mortgage rate matching this state j will influence the prepayment behavior regarding a contract issued at time t in state i. This means that the mortgage rate corresponding with state i should be derived simultaneously with the mortgage rates connected with all possible states j. The results are shown in Figure 5.12, which compares the endogenous relations based on the suboptimal prepayment rule with those of the optimal rule. Regardless of the underlying interest rate process, Figure 5.12 shows that the suboptimal prepayment rule results in mortgage rates which are lower than when the optimal rule is utilized. The reason for this is straightforward. The bank can settle for lower contract rates if borrowers prepay suboptimally.

When the CIR model underlies the valuation model, both prepayment rules result in a linear relation between the mortgage rate and the short-term interest rate. The functional relation proceeding from the suboptimal rule is not only below the one arising from the optimal prepayment rule, it is also less steep. For moderate and high interest

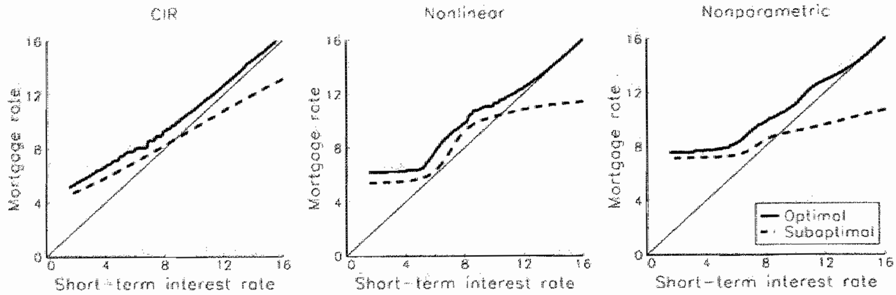


FIGURE 5.12: ENDOGENOUS MORTGAGE RATES

The mortgage rates plotted in this figure are endogenously derived such that the value of the mortgage contract equals the face value of the loan. The results are shown for both the optimal and suboptimal prepayment rule.

rates this also holds when the nonlinear or nonparametric interest rate process underlies the valuation algorithm. To explain this, we must look at the interest rate which determines the prepayment behavior. As mentioned earlier, the prepayment boundary under the optimal rule is determined by the spot rate, while the prepayment behavior under the suboptimal rule is based on the prevailing mortgage rate. This distinction is especially important when the underlying interest rate process has a strong mean-reverting drift and the short-term interest rate is outside the middle range. For example, high short-term interest rates are firmly pulled back into the middle range by both the nonlinear and nonparametric model. Since the prepayment behavior modelled by the *optimal* rule is based on this short-term interest rate, there is a substantial probability that the mortgage will be prepaid. The lender wants to be reimbursed for this prepayment risk and requires a relatively high mortgage rate. When we start with lower short-term interest rates, the prepayment likelihood drops and the contract rate required by the mortgagee decreases. At first the mortgage rate decreases at the same rate as the short-term interest rate. Later, the mortgage rate decreases less rapidly than the short-term interest rate.

When the nonlinear or nonparametric model is used, Figure 5.12 shows that the mortgage rate hardly decreases any further once the short-term interest rate is below 5 percent. At those low rates, both models have a strong upward drift which causes the discount factor to rise. The spread between the short-term interest rate and the long-term mortgage rate refers, at those levels, to the expected increase in the discount factor, and somewhat less to the prepayment risk faced by the mortgagee. This holds for endogenously derived relations based on both the optimal and suboptimal prepayment rule.

At higher rates, on the other hand, a discrepancy occurs between the two prepayment methods. As opposed to the relation proceeding from the optimal prepayment rule, the relation between both interest rates levels out when the suboptimal rule is used in combination with the nonlinear or nonparametric model. This time the prepayment decision is based on the prevailing mortgage rate rather than on the short-term interest rate. Rapidly decreasing short-term interest rates have therefore a much less pronounced effect on the prepayment behavior than when the optimal prepayment rule is applied. The flat relation between the mortgage rate and high short-term interest rates prevent the prepayment activity from increasing rapidly when the short-term interest rate decreases. Despite this limited effect on the prepayment behavior, the mortgage value alters due to the decreasing discount factors. In other words, the mortgage value increases when the short-term interest rate decreases, while the prepayment risk faced by the lender hardly changes. Consequently, the lender can make do with lower contract rates.

TABLE 5.8: SUBOPTIMAL PREPAYMENT RESULTS

Spot Rate	4%	8%	12%
	CIR model		
Annuity value	100.19	100.90	101.93
Duration annuity	50.54	48.70	46.81
Duration mortgage	41.41	15.65	6.60
Δ annuity	2.19	2.08	1.97
Δ mortgage	1.43	1.32	1.19
	Nonlinear model		
Annuity value	100.00	103.59	101.33
Duration annuity	50.84	48.07	46.70
Duration mortgage	50.84	14.48	10.64
Δ annuity	0.26	3.96	0.74
Δ mortgage	0.26	1.30	0.50
	Nonparametric model		
Annuity value	100.00	101.83	102.03
Duration annuity	49.44	48.59	47.83
Duration mortgage	49.44	10.64	6.46
Δ annuity	0.24	1.43	0.86
Δ mortgage	0.24	0.89	0.61

The table shows the valuation results for an annuity and a mortgage contract which will be prepaid when this reduces the future costs for the borrower. Given the interest rate dynamics implied in the underlying interest rate model, the mortgage rate is endogenously derived such that the value of the mortgage contract at origination equals the face value of the loan. Hence, the mortgage value is 100 by design.

The fact that the functional relation between the mortgage rate and the short-term interest rate levels out when the suboptimal prepayment rule is applied, is also reflected in the valuation results presented in Table 5.8. The results in that table are based on the endogenous relation as derived under the suboptimal prepayment rule. This is in contrast to Table 5.7, where the applied mortgage rates are endogenously derived given the optimal prepayment rule. However, the results in both tables are based on the same valuation algorithm which assumes that the mortgage is refinanced as soon as this reduces the future costs for the borrower. The differences between both tables can best be explained with the help of Figure 5.12. For example, the left diagram of Figure 5.12 illustrates that in the CIR setting, the endogenous relation between the mortgage rate and the short-term interest rate is flatter when the suboptimal rule is used rather than the optimal rule. This results in lower initial contract rates and lower prepayment probabilities as shown by the higher duration measures. When the nonlinear or nonparametric model is utilized we have to distinguish between low, moderate and high interest rates. At low rates, both the optimal and suboptimal prepayment rules result in an almost flat relation between the mortgage rate and the short-term interest rate. The prepayment likelihood is therefore very similar in both settings. As a consequence the duration measures hardly differ. At an initial short-term interest rate of 8% we see that the tangent line of the endogenous relation based on the suboptimal prepayment rule is steeper than the tangent line of the relation based on the optimal rule. Consequently, the duration measure presented in Table 5.8 is shorter than its counterpart in Table 5.5. Since the relation based on the suboptimal prepayment rule levels out, the opposite holds at an initial short-term interest rate of 12%.

To summarize, the functional relation between the mortgage rate and the short-term interest rate depends substantially on the underlying prepayment rule. As Tables 5.5, 5.7 and 5.8 show, the choice for a particular endogenous specification of the relation between the mortgage rate and short-term interest rate influences the valuation results. Especially the interest rate risk measures are found to be sensitive to this choice.

5.10 Conclusion

This chapter considers the impact alternative prepayment rules have on mortgage pricing. We distinguish between an optimal prepayment rule and a suboptimal rule. Under the optimal rule, mortgages are prepaid when the value of the mortgage, if left uncalled, exceeds the outstanding debt plus any transaction costs associated with refinancing the loan. The corresponding prepayment behavior is entirely determined by the dynamic process of the discount rate. This is in sharp contrast to the suboptimal moneyness boundary which states that the mortgage is prepaid as soon as this reduces the future costs for the borrower. This rule depends explicitly on the mortgage rate at which the individual can refinance his loan in the market.

The second topic this chapter concentrates on is the effect of alternative interest rate processes on the valuation results. For this purpose we specify three empirical one-factor models. Alongside the widely-applied Cox, Ingersoll and Ross model, we use a nonlinear and a nonparametric model. We find that the choice for a specific interest rate process influences the valuation results. In particular the duration and the effective duration Δ, which both measure the interest rate sensitivity of the contract, are sensitive to the underlying interest rate process.

Another important component of a mortgage valuation model is the function which relates the mortgage rate to the short-term interest rate. Optimal prepayment rules can only be used when this relation is endogenously derived, while the moneyness boundary allows exogenously specified relations between both interest rates. The exogenous relations used in this chapter are based on historical observations in the Netherlands and the US. The results, however, indicate that these empirical relations have serious drawbacks when they are used to value a contract not yet issued on the Dutch mortgage market. We therefore also endogenously determine the relation between the mortgage rate and the short-term interest rate for the suboptimal prepayment rule. We find that the exact shape of the endogenous relation depends not only on the applied prepayment rule but also on the underlying interest rate process. Different combinations of prepayment rules and underlying interest rate processes yield different valuation results. Again, we find that the interest rate risk measures are especially sensitive to this.

Appendix 5.A: Duration and "life expectations" of a mortgage contract

In this appendix we present an alternative way to calculate the duration of a mortgage contract. The techniques applied here to determine the duration of a callable mortgage contract are also utilized to calculate the probability that the mortgage has not been prepaid at a specific age of the contract.

To determine the duration we need to know the present value of the *expected* cash flow in each month. Given the prepayment behavior described in Section 5.3, the *conditional* cash flows in each node (i, t) can take two values:

$$C_{it} = \begin{cases} M & \text{if } \; z_t < k_t \\ M + U_t & \text{if } \; z_t \geq k_t \end{cases} \tag{5.25}$$

where z_t reflects the state of the economy at time t, *i.e.* $z_t \in \{i, 1 \leq i \leq N\}$ and k_t is the critical node at time t. If the interest rate process moves below this critical node, the mortgage will be prepaid. Note that interest rates are ordered from high to low: $r_i > r_{i+1}$, so $r_i < r_{k_t}$ if $i > k_t$. The first line in Equation (5.25) reflects the regular monthly payments. The second line displays the prepayment behavior.

The cash flows described in Equation (5.25) are *conditional* on the mortgage not having been prepaid in an earlier stage. To derive the present value of the *expected* cash flow at time t we must therefore use a discount factor which takes the possibility that the mortgage has already been prepaid into account. For this we introduce an elementary security that pays one if state i occurs at time t conditional on the interest rate path having been above the critical boundary at all times $s < t$. The valuation of such a security proceeds analogous to the derivation of state prices for Arrow-Debreu securities (see Duffie, 1996). We illustrate the procedure using Figure 5.13.

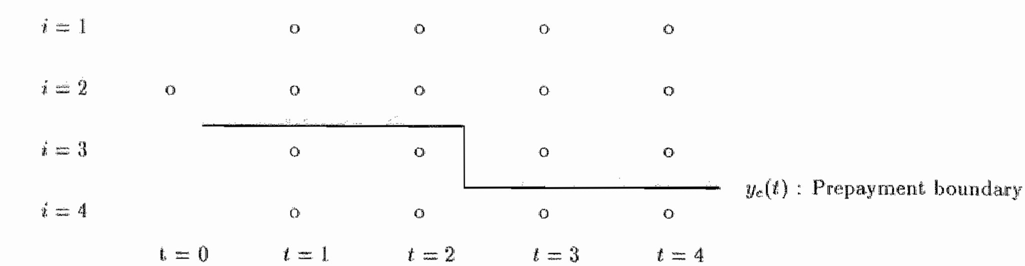


FIGURE 5.13: INTEREST RATE GRID

Consider the valuation of a security x_{34} that pays 1 in state 3 at time 4, conditional on the interest rate having always been above the prepayment boundary. Its value at each of the states at time 3 is given by

$$\begin{aligned} h_{13} &= \frac{\tilde{a}_{13}}{1+r_1} \equiv b_{13}, \\ h_{23} &= \frac{\tilde{a}_{23}}{1+r_2} \equiv b_{23}, \\ h_{33} &= \frac{\tilde{a}_{33}}{1+r_3} \equiv b_{33}, \\ h_{43} &= 0 \equiv b_{43}, \end{aligned} \tag{5.26}$$

where $\tilde{a}_{ij}$ are the risk-neutral transition probabilities of the Markov chain. The elements $b_{ij} = \frac{\tilde{a}_{ij}}{(1+r_i)}$ are discounted probabilities and stored in the matrix $\mathbf{B}$, as discussed in Section 5.3. Continuing the pricing algorithm, the value of the elementary security at time 2 follows as:

$$h_{i2} = \begin{cases} \sum_{j=1}^{3} b_{ij} h_{j3} & i = 1, 2, \\ 0 & i = 3, 4, \end{cases} \tag{5.27}$$

and for $t = 1$:

$$h_{i1} = \begin{cases} \sum_{j=1}^{2} b_{ij} h_{j2} & i = 1, 2, \\ 0 & i = 3, 4. \end{cases} \tag{5.28}$$

Finally the "*conditional state price*" q_{34} can be calculated as the value of this security at time $t = 0$. Since the tree starts from the initial condition $z_0 = 2$, we have

$$q_{34} = \sum_{j=1}^{2} b_{2j} h_{j1}. \tag{5.29}$$

To derive the general formula for all conditional state prices q_{it} $(i = 1, \ldots, N, t = 1, \ldots, T)$, first consider pricing all elementary securities x_{i4} at time $t = 4$. Going backwards, their value at time $t = 3$ can be stored in the (4×4) matrix $\mathbf{H}_3$, which is given by

$$\mathbf{H}_3 = \mathbf{J}_3 \mathbf{B}, \tag{5.30}$$

where $\mathbf{J}_3$ is the selection matrix that takes the first k_3 rows from $\mathbf{B}$, and sets the remaining rows equal to zero,

$$\mathbf{J}_3 = \begin{pmatrix} \mathbf{I}_{k_3} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}, \tag{5.31}$$

where $\mathbf{I}_{k_3}$ is the identity matrix of order k_3. The vector of state prices $q_4 = (q_{14}, \ldots, q_{44})'$ is found by the backward recursion illustrated above,

$$\mathbf{q}_4' = \mathbf{e}_2' \mathbf{B} \left(\prod_{s=1}^{3} \mathbf{J}_s \mathbf{B} \right), \tag{5.32}$$

where $\mathbf{e}_2$ is the vector $(0, 1, 0, 0)'$, and all $\mathbf{J}_s$ are defined analogous to the selection matrix $\mathbf{J}_3$ above, selecting the first k_s rows from $\mathbf{B}$ and filling the remaining rows with zeros. In general, in a tree with N states, the vector of state prices for period t can be written as:

$$\mathbf{q}'_t = \mathbf{e}'_{z_0}\mathbf{B}\left(\prod_{s=1}^{t-1}\mathbf{J}_s\mathbf{B}\right), \tag{5.33}$$

where $\mathbf{e}_{z_0}$ is a $(N \times 1)$ vector with a one in position z_0 and zeros elsewhere. Equation (5.33) can be transformed to the forward recursion to derive conditional state prices:

$$\begin{aligned} \mathbf{q}'_{t+1} &= \mathbf{e}'_{z_0}\mathbf{B}\left(\prod_{s=1}^{t}\mathbf{J}_s\mathbf{B}\right), \\ &= \mathbf{q}'_t\mathbf{J}_t\mathbf{B}. \end{aligned} \tag{5.34}$$

Equation (5.34) defines a forward recursion for the valuation of the conditional cash flows C_{it} in state i at time t. These conditional state prices $\mathbf{q}_t$ can be used to determine the value and the duration a mortgage contract. For example, the value of the mortgage in state j at time 0, represented by v_{j0}, is defined as the present value of the future cash flows:

$$v_{j0} = \sum_{t=1}^{T}\sum_{i=1}^{N} C_{it}q_{it}. \tag{5.35}$$

Note that we hereby assume that the critical boundary is known. Based on this setup we define duration as:

$$D_{j0} = \frac{1}{v_{j0}}\sum_{t=1}^{T}\sum_{i=1}^{N} tC_{it}q_{it}, \tag{5.36}$$

where D_{j0} is the duration of a mortgage whose contract rate y_j is based on the short-term interest rate r_j, and v_{j0} is the corresponding mortgage value. Furthermore, C_{it} is the conditional cash flow at node (i, t) of the tree and q_{it} is the conditional state price at that node corresponding to the risk-neutral probabilities. Note that both C_{it} and q_{it} depend on state j at time $t = 0$.

Similar to the derivation of the conditional state prices we can determine the probability that a mortgage has not been prepaid at a specific age of the contract. For this we have to replace the matrix $\mathbf{B}$ in Equation (5.34) by the matrix $\mathbf{A}$ which contains the transition probabilities of the underlying interest rate process. Since we are calculating the "life expectations" of the mortgage rather than its value we must use the matrix $\mathbf{A}$ instead of its risk-neutral counterpart $\tilde{\mathbf{A}}$. Hence, the probability that the mortgage has not been prepaid at time t can be calculated by the forward recursion:

$$\mathbf{l}'_t = \mathbf{l}'_{t-1}\mathbf{J}_{t-1}\mathbf{A}, \tag{5.37}$$

where $\mathbf{l}_t$ is an $(n \times 1)$ vector with an element l_{it} being the probability that interest rate process arrives in node i at time t and that the followed interest rate path has been above the critical boundary at all times $s < t$. In other words, the mortgage has not been prepaid in an earlier stage. The matrix $\mathbf{J}_{t-1}$ is defined analogous to the selection matrix $\mathbf{J}_3$ in Equation (5.31).

Appendix 5.B: The Markov transition matrix for the CIR model

In this appendix the explicit finite difference method will be used to construct a Markov transition matrix which describes the short-term interest rate process based on the CIR interest rate model:

$$dr = \kappa(\theta - r(t))dt + \sigma\sqrt{r(t)}dz. \tag{5.38}$$

This appendix borrows heavily from Hull and White (1990a, 1990b, 1993). The transformation described here can also be used for the Vasicek model, however, in that case there is no reason to apply the change of variable technique.

Instead of using the short-term interest rate as the state variable, Hull and White define a new state variable $x(r,t)$, which has a constant instantaneous standard deviation. From Ito's Lemma we know that the process followed by $x(r,t)$ can be described as:

$$dx = \left(\mu(r,t)\frac{\partial x}{\partial r} + \frac{\partial x}{\partial t} + \frac{1}{2}\frac{\partial^2 x}{\partial r^2}\sigma^2(r,t)\right)dt + \sigma(r,t)\frac{\partial x}{\partial r}dz, \tag{5.39}$$

where $\mu(r,t)$ and $\sigma(r,t)$ are the drift and volatility, respectively, as defined in the CIR model. We wish to find a transformation from r, the interest rate, to the new state variable, x, that has a constant volatility, therefore:

$$\sigma(r,t)\frac{\partial x}{\partial r} = v, \tag{5.40}$$

where v is a constant and $\sigma(r,t) = \sigma\sqrt{r(t)}$ as in the CIR model. For the standard deviation to be constant, the stochastic process $x(r,t)$ has to be a linear function of $\sqrt{r(t)}$:

$$x(r,t) = \frac{2v}{\sigma}\sqrt{r(t)}. \tag{5.41}$$

If we choose $v = \frac{1}{2}\sigma$ then $x(r,t) = \sqrt{r(t)}$. The process described by Equation (5.39) can now be written as :

$$dx = gdt + vdz, \tag{5.42}$$

where

$$\begin{aligned} v &= \frac{1}{2}\sigma, & (5.43)\\ g &= \mu(r,t)\frac{\partial x}{\partial r} + \frac{\partial x}{\partial t} + \frac{1}{2}\frac{\partial^2 x}{\partial r^2}\sigma^2(r,t), & (5.44)\\ &= \frac{\alpha_1}{x} - \alpha_2 x, \end{aligned}$$

with $\alpha_1 = \frac{4\kappa\theta - \sigma^2}{8}$ and $\alpha_2 = \frac{1}{2}\kappa$.

The discretization of this stochastic process can proceed in two alternative ways (see Figure 5.14). The first is the explicit finite difference method, this method relates the value at time t to three values at time $t + \Delta t$. On the other hand, the implicit finite difference method relates the value at time t to three values at time $t - \Delta t$.

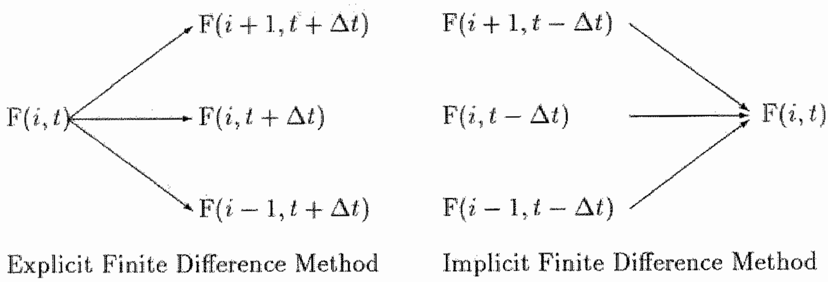


FIGURE 5.14: FINITE DIFFERENCE METHOD

In both cases the value at a specific time is related to *three* values at time $t + \Delta t$ (explicit) or at time $t - \Delta t$ (implicit). This is done because, as shown by Brennan and Schwartz (1978), the explicit finite difference method is equivalent to a trinomial lattice approach, while the implicit finite difference method is equivalent to a multinomial lattice approach.

To model the interest rate dynamics we use the explicit finite difference method. The results from this lattice or tree approach will be combined in a Markov chain model of interest rates. In order to be sure that the Markov process yields a similar unconditional density distribution as the original continuous time model, the first and second moments of each model have to be equal in the limit $\Delta t \to 0$. This means that for a mean-reverting interest process we can no longer insist that the process moves from state i to one of $i + 1$, i or $i - 1$ as was graphically represented in Figure 5.14. By introducing an integer m, we allow bigger fluctuations towards $m + 1$, m or $m - 1$, where m may or may not be equal to i. The alternative branching possibilities are shown in Figure 5.15.

It can be shown that in Figure 5.15 $i = m$ when:

$$-\frac{1}{2} \leq \left(\frac{\alpha_1}{x} - \alpha_2 x\right) \frac{\Delta t}{\Delta x} \leq \frac{1}{2}, \tag{5.45}$$

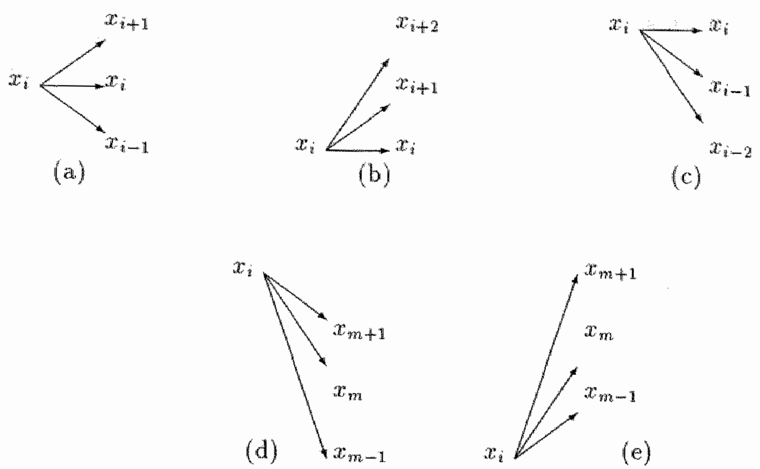
with α_1 and α_2 defined as before. If these two parameters are positive, Equation (5.45) can be written as:

$$x_{min} \leq x \leq x_{max} \tag{5.46}$$

where

$$x_{min} = \frac{1}{2\alpha_2}\left(-\beta + \sqrt{\beta^2 + 4\alpha_1\alpha_2}\right), \tag{5.47}$$

$$x_{max} = \frac{1}{2\alpha_2}\left(\beta + \sqrt{\beta^2 + 4\alpha_1\alpha_2}\right), \tag{5.48}$$



Source: Hull and White (1990a)

FIGURE 5.15: MEAN-REVERTING POSSIBILITIES

$$\beta = \frac{\Delta x}{2\Delta t}. \tag{5.49}$$

If $x < x_{min}$, the tree from Figure 5.15b will be used. And if $x > x_{max}$, Figure 5.15c will be chosen. By using these two trees when x is imminent to become either too small or too large, g is bounded and the estimated value converges to its true value.

The integer m has to be chosen such that the first and second moments of the change in x are correct during the time interval $\Delta t \to 0$. In this limit $\Delta t \to 0$ the discrete approximation becomes its own original continuous time model. The first and second moment will be accurate if the following equations are satisfied:

$$p_{i,m-1}(m-1)\Delta x + p_{i,m} m \Delta x + p_{i,m+1}(m+1)\Delta x = E(x), \tag{5.50}$$

$$p_{i,m-1}(m-1)^2\Delta x^2 + p_{i,m} m^2 \Delta x^2 + p_{i,m+1}(m+1)^2\Delta x^2 = v^2\Delta t + E(x)^2, \tag{5.51}$$

$$p_{i,m-1} + p_{i,m} + p_{i,m+1} = 1, \tag{5.52}$$

where $E(x) = g$ is the drift rate after changing the state variable and $p_{i,j}$ represents the probability that the process moves from state i to state j in the time interval Δt. As presented by Hull and White (1990) the solution of these equations is:

$$p_{i,m-1} = \frac{1}{2}\left(m^2 + m - (1+2m)\frac{E(x)}{\Delta x} + \frac{E(x)^2}{\Delta x^2} + \frac{v^2\Delta t}{\Delta x^2}\right), \tag{5.53}$$

$$p_{i,m} = 1 - m^2 + \frac{2mE(x)}{\Delta x} - \frac{E(x)^2}{\Delta x^2} - \frac{v^2\Delta t}{\Delta x^2}, \tag{5.54}$$

$$p_{i,m+1} = \frac{1}{2}\left(m^2 - m + (1-2m)\frac{E(x)}{\Delta x} + \frac{E(x)^2}{\Delta x^2} + \frac{v^2\Delta t}{\Delta x^2}\right), \tag{5.55}$$

hereby is noted that the explicit finite difference method provides one degree of freedom: the choice of $\frac{v^2\Delta t}{\Delta x^2}$. However, the choice for this ratio is not completely without bounds. It is necessary that $p_{i,m-1}$, $p_{i,m}$ and $p_{i,m+1}$ are positive and smaller than or equal to one. This implies that $0.25 < \frac{v^2\Delta t}{\Delta x^2} < 0.75$.

Proof: If we look at one time interval Δt, the interest process can move from state x_i to one of x_1, x_2 or x_3 with probabilities $p_{i,1}$, $p_{i,2}$ and $p_{i,3}$, respectively, where the subscripts 1, 2 and 3 are randomly chosen, they can represent each possible branching process from Figure 5.15. The relations between x_1, x_2 and x_3 are $x_1 = x_2 + \Delta x$ and $x_3 = x_2 - \Delta x$, so that $E(x) = x_2 + \varpi\Delta x$ where $-1 \leq \varpi \leq 1$. The first moment condition demands that

$$p_{i,1}x_1 + p_{i,2}x_2 + p_{i,3}x_3 = x_2 + \varpi\Delta x. \tag{5.56}$$

Taking into account that the probabilities have to sum to one, the first moment condition tells us that $p_{i,1} - p_{i,3} = \varpi$. The second moment constraint requires that

$$p_{i,1}x_1^2 + p_{i,2}x_2^2 + p_{i,3}x_3^2 = v^2\Delta t + (x_2 + \varpi\Delta x)^2. \tag{5.57}$$

Combining this second moment constraint with the relations between x_1, x_2 and x_3 yields:

$$\begin{aligned} p_{i,1} &= \tfrac{1}{2}(\varpi^2 + \varpi + \tfrac{v^2\Delta t}{\Delta x^2}), \\ p_{i,2} &= 1 - \varpi^2 - \tfrac{v^2\Delta t}{\Delta x^2}, \\ p_{i,3} &= \tfrac{1}{2}(\varpi^2 - \varpi + \tfrac{v^2\Delta t}{\Delta x^2}). \end{aligned} \tag{5.58}$$

The probabilities $p_{i,1}$ and $p_{i,3}$ are minimal when ϖ is $-\frac{1}{2}$ and $\frac{1}{2}$, respectively, while $p_{i,2}$ has its maximum value when $\varpi = 0$. These values for ϖ result in the following constraints for the different probabilities:

	$\varpi = -\frac{1}{2}$	$\varpi = 0$	$\varpi = \frac{1}{2}$
$p_{i,1}$	$\frac{v^2\Delta t}{\Delta x^2} \geq \frac{1}{4}$	$0 \leq \frac{v^2\Delta t}{\Delta x^2} \leq 2$	$0 \leq \frac{v^2\Delta t}{\Delta x^2} \leq \frac{5}{4}$
$p_{i,2}$	$0 \leq \frac{v^2\Delta t}{\Delta x^2} \leq \frac{3}{4}$	$0 \leq \frac{v^2\Delta t}{\Delta x^2} \leq 1$	$\frac{v^2\Delta t}{\Delta x^2} \geq \frac{1}{4}$
$p_{i,3}$	$0 \leq \frac{v^2\Delta t}{\Delta x^2} \leq \frac{5}{4}$	$0 \leq \frac{v^2\Delta t}{\Delta x^2} \leq 2$	$0 \leq \frac{v^2\Delta t}{\Delta x^2} \leq \frac{3}{4}$

Taking the relevant constraints we find that $\frac{1}{4} \leq \frac{v^2\Delta t}{\Delta x^2} \leq \frac{3}{4}$. Q.E.D.•

Hull and White suggest a value of $\frac{1}{3}$ for $\frac{v^2\Delta t}{\Delta x^2}$ which means that $\Delta x = v\sqrt{3\Delta t}$. The constraint that just has been derived requires $\frac{1}{2}\Delta x \leq v\sqrt{\Delta t} \leq \frac{1}{2}\sqrt{3}\Delta x$ which means that $\frac{2v\sqrt{\Delta t}}{\sqrt{3}} \leq \Delta x \leq 2v\sqrt{\Delta t}$. Hull and White's choice satisfies this condition. We will use the same value for $\frac{v^2\Delta t}{\Delta x^2}$ to define a finite state Markov chain with Equation (5.46) indicating the borders. These boundaries, together with the transform probabilities as defined in Equations (5.53), (5.54) and (5.55) describe the Markov process.

Chapter 6

A VAR analysis of interest rates in the Netherlands

6.1 Introduction

Mortgage valuation models are commonly based on one-factor interest rate models. For example, Dunn and McConnell (1981a,b), Hendershott and Van Order (1987) and Kau, Keenan, Muller and Epperson (1990) all use the Cox, Ingersoll and Ross (1985b) square root process to model the term structure. The effect of alternative one-factor interest rate models on the value and risk characteristics of mortgages was studied in the previous chapter.

Various authors argue that a single factor model is not sufficient to describe the entire term structure. For example, Longstaff and Schwartz (1992) advocate a two-factor model which includes the spot rate volatility along the short-term interest rate. Brennan and Schwartz (1985) include the long-term yield as a second factor. In keeping with this, Schwartz and Torous (1989) and Boudoukh, Richardson, Stanton and Whitelaw (1997) base their mortgage pricing models on two factors and find that introducing the long-term interest rate as a second variable improves the valuation results for US Mortgage-Backed Securities.

Alongside both term structure variables, a third factor related to the mortgage market might be required to model the mortgage rate dynamics accurately. This assumption appears reasonable for countries without a well-developed secondary market in Mortgage-Backed Securities like the Netherlands.

This chapter addresses the question how many factors are needed to describe the mortgage rate dynamics correctly. For this we study the empirical relation between the one-month interest rate, the long-term interest rate and the mortgage rate in the Netherlands.

Vector AutoRegressive (VAR) techniques are used to analyze the dynamic interactions between the variables. This chapter presents the results for both a stationary and a unit root specification of the VAR model. The former specification is consistent with mean-reverting interest rate models, the latter models a random walk.

This chapter is organized as follows: In Section 6.2 the vector autoregressive technique is introduced and briefly explained. Section 6.3 describes the interest rate data used in this paper. The actual analysis is done in Section 6.4. First, the number of lags included in the VAR analyses is determined in Subsection 6.4.1. Granger causality tests are applied in Subsection 6.4.2, and impulse response functions and variance decompositions are estimated in Subsections 6.4.3 and 6.4.4. The main conclusions are summarized in Section 6.5.

6.2 Methodology

In order to analyze the relations between the short-term interest rate, the long-term interest rate and the mortgage rate in the Netherlands a Vector AutoRegressive (VAR) model is utilized. A VAR model, as introduced by Sims (1980), is well suited for analyzing the joint linear dynamics in a system of time series. Within a vector autoregressive model, variables are regressed on a constant and p of their own lags, as well as on p lags of all other variables in the system. A p^{th} order vector autoregression, denoted by VAR(p), is a vector generalization of a p^{th} order autoregression, AR(p), and reads:

$$\mathbf{x}_t = \mathbf{c} + \mathbf{\Phi}_1\mathbf{x}_{t-1} + \mathbf{\Phi}_2\mathbf{x}_{t-1} + \ldots + \mathbf{\Phi}_p\mathbf{x}_{t-p} + \epsilon_t. \tag{6.1}$$

The vector $\mathbf{x}_t$ contains the observed values at time t of n variables. Furthermore, $\mathbf{c}$ is a $(n \times 1)$ vector of constants, and $\mathbf{\Phi}_j$ denotes a $(n \times n)$ matrix of autoregressive coefficients for $j = 1, 2, \ldots p$. The error terms in Equation (6.1), ϵ_t, are serially uncorrelated, with mean zero, and covariance matrix $\mathbf{\Sigma}$. Without restrictions on the parameters, this vector autoregression can be estimated with n Ordinary Least Squares regressions where each regression has the same set of explanatory variables.[1]

The validity of the VAR analysis depends on the stationarity of the time series. If a time series has a unit root, one can take first differences before including it in the VAR model. Alternatively, if two unit root processes are cointegrated, a stationary linear combination of these variables can be found which can be included in the VAR model.[2] See Engle and Granger (1987), Johansen (1988) and Hamilton (1994) for a discussion on cointegration and unit roots. Campbell and Perron (1991) is a useful, non-technical primer on these issues.

In this chapter, we do not report the estimated VAR parameters because the linear dynamics of a VAR model are better illustrated with impulse response functions and variance decompositions.

An impulse response function describes the reaction of all included variables to a shock in one of them. To derive the impulse response function it is helpful to first rewrite the

[1] For a more detailed explanation of vector autoregressions see Hamilton (1994).

[2] The cointegrating vectors of the interest rates analyzed in this chapter are of the form (1,-1), such that the spreads are stationary. The permanent effect of a shock in one of the variables is therefore the same for each variable.

VAR(p) process in its companion form.[3] Suppressing constants the companion form reads:

$$\begin{aligned} \boldsymbol{\xi}_t &= \mathbf{F}\boldsymbol{\xi}_{t-1} + \mathbf{G}\boldsymbol{\varepsilon}_t, \\ \mathbf{x}_t &= \mathbf{H}\boldsymbol{\xi}_t, \end{aligned} \tag{6.2}$$

where

$$\underset{(np\times 1)}{\boldsymbol{\xi}_t} \equiv \begin{pmatrix} \mathbf{x}_t \\ \mathbf{x}_{t-1} \\ \vdots \\ \mathbf{x}_{t-p+1} \end{pmatrix}, \tag{6.3}$$

$$\underset{(np\times np)}{\mathbf{F}} \equiv \begin{pmatrix} \boldsymbol{\Phi}_1 & \boldsymbol{\Phi}_2 & \boldsymbol{\Phi}_3 & \cdots & \boldsymbol{\Phi}_{p-1} & \boldsymbol{\Phi}_p \\ \mathbf{I}_n & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_n & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I}_n & \mathbf{0} \end{pmatrix}, \tag{6.4}$$

$$\underset{(np\times n)}{\mathbf{G}} \equiv \begin{pmatrix} \mathbf{I} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{pmatrix}, \tag{6.5}$$

$$\underset{(n\times np)}{\mathbf{H}} \equiv \begin{pmatrix} \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} \end{pmatrix}, \tag{6.6}$$

and ε_t is an $(n \times 1)$ vector of residuals. By recursive substitution, Equation (6.2) can be brought to:

$$\boldsymbol{\xi}_{t+s} = \mathbf{G}\boldsymbol{\varepsilon}_{t+s} + \mathbf{F}\mathbf{G}\boldsymbol{\varepsilon}_{t+s-1} + \mathbf{F}^2\mathbf{G}\boldsymbol{\varepsilon}_{t+s-2} + \ldots + \mathbf{F}^{s-1}\mathbf{G}\boldsymbol{\varepsilon}_{t+1} + \mathbf{F}^s\boldsymbol{\xi}_t. \tag{6.7}$$

If the eigenvalues of $\mathbf{F}$ are all inside the unit circle, then $\lim_{s\to\infty} \mathbf{F}^s \to \mathbf{0}$ and the VAR model can be written as an infinite order Moving Average process:

$$\begin{aligned} \mathbf{x}_t &= \boldsymbol{\varepsilon}_t + \tilde{\boldsymbol{\Psi}}_1\boldsymbol{\varepsilon}_{t-1} + \tilde{\boldsymbol{\Psi}}_2\boldsymbol{\varepsilon}_{t-2} + \tilde{\boldsymbol{\Psi}}_3\boldsymbol{\varepsilon}_{t-3} + \ldots & (6.8) \\ &= \sum_{s=0}^{\infty} \tilde{\boldsymbol{\Psi}}_s\boldsymbol{\varepsilon}_{t-s}, & (6.9) \\ &= \sum_{s=0}^{\infty} \left(\tilde{\boldsymbol{\Psi}}_s\mathbf{L}\right)\boldsymbol{\eta}_{t-s}, & (6.10) \\ &= \sum_{s=0}^{\infty} \boldsymbol{\Psi}_s\boldsymbol{\eta}_{t-s}, & (6.11) \end{aligned}$$

[3] Before rewriting the unit root specification we imposed an additional restriction on the constant terms of the VAR model to exclude drifts in the interest rates.

where

$$
\begin{aligned}
\varepsilon_{t-s} &= \mathbf{L}\boldsymbol{\eta}_{t-s}, &(6.12)\\
\boldsymbol{\Psi}_s &= \tilde{\boldsymbol{\Psi}}_s\mathbf{L}, &(6.13)\\
E\left[\boldsymbol{\eta}_{t-s}\boldsymbol{\eta}'_{t-s}\right] &= \mathbf{I}, &(6.14)
\end{aligned}
$$

and the matrix $\mathbf{L}$ is the Choleski decomposition of the estimated covariance matrix: $\tilde{\Sigma}(p) = \mathbf{LL}'$, with $\mathbf{L}$ being a lower triangular matrix. Since $\mathbf{L}$ is a lower triangular matrix the ordering of the variables is of great importance. Changing the order results in a different $\mathbf{L}$, and therefore different impulse response functions. The dynamic multiplier $\tilde{\boldsymbol{\Psi}}_s$ is equal to $\mathbf{HF}^s\mathbf{G}$. Element (i, j) of the matrix $\boldsymbol{\Psi}_s$ is the response of the i^{th} element of $\mathbf{x}_{t+s}$ on an increase in the j^{th} element of $\boldsymbol{\eta}_t$. A plot of this element as a function of s is referred to as the *impulse response function*.

The key question that can be addressed with impulse response functions is how an unexpected shock in one variable at time t will cause us to revise the predictions for the future values of the separate variables. More formally, we are estimating:

$$
\boldsymbol{\Psi}_s(i,j) = \frac{\partial E(x_{i,t+s} \mid \boldsymbol{\eta}_{\mathbf{j,t}}, \mathbf{I_{t-1}})}{\partial \boldsymbol{\eta}_{\mathbf{j,t}}}. \tag{6.15}
$$

Here $\boldsymbol{\Psi}_s(i,j)$ is element (i,j) of the matrix $\boldsymbol{\Psi}_s$, i is the variable whose response to the unexpected change of shock j is analyzed, with $i, j \in 1, \dots n$. $\mathbf{I}_{t-1}$ is the set of information at time $t-1$ on which the expectations of $x_{j,t}$ were based. This information set does include the lagged history of x_t.

The representation of the VAR model applied in the impulse response analysis provides another way to analyze the inter-relationships within a system of variables. Given Equation (6.11), the error in forecasting a VAR model s periods-ahead can be assigned to innovations in specific variables. By decomposing the variance in this way, we get more insight into the relative importance of the random shocks.

The error of the s periods-ahead forecast in a VAR model is:

$$
\mathbf{x}_{t+s} - \hat{\mathbf{x}}_{t+s} = \varepsilon_{t+s} + \boldsymbol{\Psi}_1\varepsilon_{t+s-1} + \boldsymbol{\Psi}_2\varepsilon_{t+s-2} + \boldsymbol{\Psi}_3\varepsilon_{t+s-3} + \dots + \boldsymbol{\Psi}_{s-1}\varepsilon_{t+1} \tag{6.16}
$$

such that the Mean Squared Error of this forecast is:

$$
\begin{aligned}
\mathrm{MSE}(\hat{\mathbf{x}}_{t+s}) &= E\left[(\mathbf{x}_{t+s} - \hat{\mathbf{x}}_{t+s})(\mathbf{x}_{t+s} - \hat{\mathbf{x}}_{t+s})'\right],\\
&= \boldsymbol{\Sigma} + \boldsymbol{\Psi}_1\boldsymbol{\Sigma}\boldsymbol{\Psi}'_1 + \boldsymbol{\Psi}_2\boldsymbol{\Sigma}\boldsymbol{\Psi}'_2 + \dots + \boldsymbol{\Psi}_{s-1}\boldsymbol{\Sigma}\boldsymbol{\Psi}'_{s-1}. &(6.17)
\end{aligned}
$$

Since $\boldsymbol{\Sigma} = \mathbf{LL}'$ Equation (6.17) can be rewritten as:

$$
\begin{aligned}
\mathrm{MSE}(\hat{\mathbf{x}}_{t+s}) &= \sum_{j=1}^{n} l_j l'_j + \boldsymbol{\Psi}_1 l_j l'_j \boldsymbol{\Psi}'_1 + \boldsymbol{\Psi}_2 l_j l'_j \boldsymbol{\Psi}'_2 + \dots + \boldsymbol{\Psi}_{s-1} l_j l'_j \boldsymbol{\Psi}'_{s-1},\\
&= \sum_{t=0}^{s-1}\sum_{j=1}^{n} \boldsymbol{\Psi}_t l_j l'_j \boldsymbol{\Psi}'_t, &(6.18)
\end{aligned}
$$

where l_j is the j^{th} column of the lower triangular matrix $\mathbf{L}$. The m^{th} diagonal element of this matrix $\mathrm{MSE}(\hat{\mathbf{x}}_{t+s})$ is the variance of the s periods ahead forecast of variable m. The contribution of innovations in the k^{th} variable to this variance is given by the m^{th} diagonal element $\sum_{t=0}^{s-1} \mathbf{\Psi}_t l_k l_k' \mathbf{\Psi}_t'$. This way the forecast error variance can be decomposed into the components accounted for by shocks in the individual variables.

6.3 Description of the data

The empirical analysis in this paper is based on a system of time series of the short-term interest rate, the long-term interest rate and the mortgage rate. The period under investigation runs from January 1972 to December 1995.

From January 1972 to April 1980, the return on one-month loans to local authorities is used as the short-term interest rate.[4] From April 1980 on, the one-month Holland Interbank rate is used. Both time series are end-of-month observations and are identical during the period for which we have data on both time series. The long-term interest rate is the yield on government bonds with a remaining time to maturity of 5 to 8 years. This time to maturity resembles the period during which the contract rate of most Dutch mortgages is fixed. For the mortgage rate we consider annuity-mortgage contracts whose contract rates are fixed for 5 years while the maturities are equal to 30 years. The Holland Interbank rate, the yield on government bonds and the mortgage rates are obtained from DataStream.

The time series are plotted on the left-hand side of Figure 6.1. As can be seen from the top left drawing, the short-term interest rate time series has two outliers. In September and October 1976 the one-month interest rate reached a level of 18 percent while shortly before and after these two months the interest rate was at a more normal, and therefore lower, level. As reported by Schotman (1989), these months are known for the tumult on the European exchange markets.

Additionally, it can be deduced from the right-hand diagrams of Figure 6.1 that the yield curve was frequently inverse in the late seventies and early eighties. This phenomenon also occurred in the early nineties. In those periods with inverse term structures, the short-term interest rate was occasionally even higher than the mortgage rate. In the 24 years with monthly observations, the short-term interest rate exceeds the mortgage rate in 26 months. Especially remarkable in this respect is 1992. During most of that year, the levels of the short-term interest rate and the mortgage rate followed each other closely. The long-term interest rate, however, was often almost 1 percentage point below that level. Besides these periods with inverse term structures, we observe that in May and June 1976 the long-term interest rate was higher than the mortgage rate.

The top right corner of Figure 6.1 suggests heteroskedasticity in the volatility of the first differences of the short-term interest rate. This can result in an inconsistent estimation

[4] The time series of the return on one-month loans to local authorities has been constructed by Kool and Rouwenhorst (1985) and was analyzed in depth by Schotman (1989).

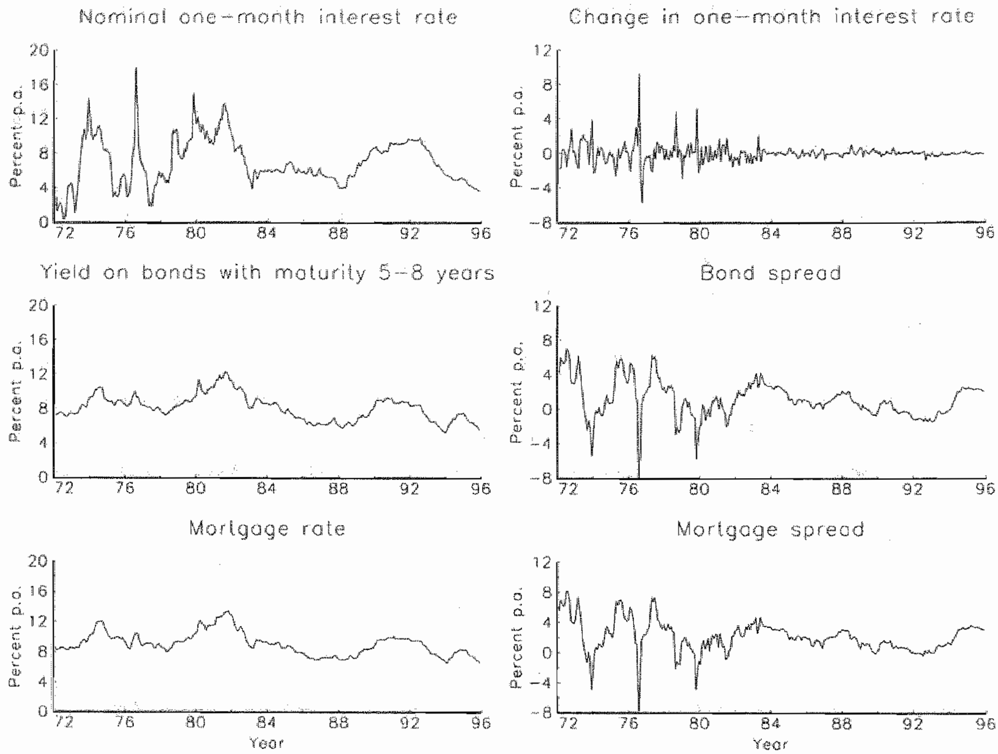


FIGURE 6.1: TIME SERIES OF INTEREST RATES

The left-hand diagrams of this figure show the short-term, long-term and mortgage interest rates, respectively. The right-hand diagrams illustrate the monthly fluctuations in the short-term interest rate, the differences between the long-term and short-term interest rates and the spread between the mortgage rate and the short-term interest rate.

TABLE 6.1: SUMMARY STATISTICS OF INTEREST RATES, JAN.1972-DEC.1995

	r	l	y	Δr	$l-r$	$y-r$
Mean	6.95	8.12	9.08	-0.01	1.17	2.13
Maximum	18.00	12.30	13.45	9.25	7.10	8.13
Minimum	0.38	5.27	6.51	-5.75	-7.97	-7.75
Stand. Dev.	2.77	1.47	1.49	1.15	2.14	2.11
Skewness	0.47	0.43	0.73	1.99	-0.07	-0.14
Kurtosis	3.49	2.86	3.24	21.96	4.54	5.37
ADF	-4.10***	-1.90	-2.02	-9.96***	-4.94***	-5.08***

The first three columns report the statistics of the short-term interest rate, the long-term interest rate and the mortgage rate, respectively. These rates are expressed in percentages per annum. The fourth column presents the statistics of the first differences of the short-term interest rate, while for the following two columns the short-term interest rate is subtracted from the long-term interest rate and the mortgage rate, respectively. The skewness of a series x_t is calculated as $SK = \frac{1}{\sigma_x^3}\frac{1}{T}\sum_{t=0}^{T}\left[(x_t-\bar{x})^3\right]$, where T is the number of observations. The thickness of the tails is measured by the kurtosis: $KU = \frac{1}{\sigma_x^4}\frac{1}{T}\sum_{t=0}^{T}\left[(x_t-\bar{x})^4\right]$. The Augmented Dickey-Fuller (ADF) test is applied to test for the null hypothesis of a unit root. One asterisk denotes significance at a 10% level, two asterisks at a 5% level and three at a 1% level.

of the standard errors of the parameters in the VAR system. In order to correct for this a White procedure is applied.

The bottom right corner of Figure 6.1 shows the historical spread between the mortgage rate and the short-term interest rates in the Netherlands. For the period 1981-1994, this spread is compared with its American counterpart in Figure 5.3 of Section 5.5. There we also present the summary statistics of this spread for both countries.

Table 6.1 provides descriptive statistics for each of the series. In this paper the Augmented Dickey-Fuller (ADF) test is applied to test for the null hypothesis of a unit root.[5] If the absolute value of the ADF-statistic is larger than the critical value the hypothesis of a unit root can be rejected. If we concentrate on the level of the interest rates, Table 6.1 shows that the non-stationarity hypothesis can be rejected for the one-month interest rate, but not for the other interest rates. Table 6.1 also reports the ADF-statistics of time series after transformation. As shown by Campbell and Shiller (1987), simple first differencing of all variables can lead to econometric problems. Hence, we utilize the spreads between the different yields rather than the first differences of the long-term interest rate and mortgage rate. This assumes that the levels of y and r are cointegrated.

The unit root tests summarized by the ADF-statistics in Table 6.1 are inconsistent with each other. A linear transformation of two stationary processes has to be stationary as well. Since we find that the short-term interest rate is a stationary process, just like the

[5] Figure 6.1 suggest that structural breaks may have occured in both the interest rate levels and spreads. This might bias the ADF test. See Perron (1989, 1993) for a discussion on this topic.

spread between the different yields and the short-term interest rate, we would expect the different yields to be stationary as well. This does not conform with the results in Table 6.1, however. Unit roots tests like the Augmented Dickey-Fuller test have frequently been questioned in the literature. For example, Cochrane (1991) warns against an excessively strict application of these kinds of tests such that we have to be careful with drawing strong conclusions based on the ADF statistics.

TABLE 6.2: CORRELATION MATRIX OF INTEREST RATES, JAN.1972-DEC.1995

	r	Δr	l	Δl	y	Δy	$l-r$	$\Delta(l-r)$	$y-r$
r	1								
Δr	0.20	1							
l	0.64	-0.00	1						
Δl	0.16	0.31	0.12	1					
y	0.66	-0.06	0.97	-0.01	1				
Δy	0.24	0.14	0.18	0.57	0.12	1			
$l-r$	-0.85	-0.27	-0.15	-0.12	-0.19	-0.19	1		
$\Delta(l-r)$	-0.17	-0.97	0.03	-0.07	0.06	0.00	0.25	1	
$y-r$	-0.85	-0.31	-0.16	-0.21	-0.16	-0.23	0.99	0.27	1
$\Delta(y-r)$	-0.15	-0.97	0.04	-0.18	0.09	0.09	0.23	0.98	0.26

The table reports the correlation coefficients between the interest rates in different specifications.

Table 6.2 reports the correlation coefficients between the variables, their first differences and the spreads between the long and short-term interest rate and the mortgage rate and short-term interest rate. Table 6.2 illustrates the high correlation between the level of the short-term interest rate, the long-term interest rate and the mortgage rate. It especially appears that the long-term interest rate and the mortgage rate follow each other closely. When looking at the first differences we see that the correlations of the short-term interest rate with both the long-term interest rate and mortgage rate decrease considerably, while the correlation coefficient of the long-term interest rate and mortgage rate remains high. Table 6.2 therefore indicates that monthly fluctuations of the long-term interest rates and mortgage rates are more or less independent of changes in the short-term interest rate. This suggests that a second factor is needed to model the mortgage rate dynamics accurately.

As there is some doubt about the stationarity of a levels specification, we analyze a VAR model in levels as well as a cointegrated system. In the first specification we include the level of the short-term interest rate and the spread between the yield on government bonds with a maturity of 5 to 8 years and the short-term interest rate, as well as the spread between the mortgage rate and the short-term interest rate. We refer to this VAR model as the *stationary specification*. This specification corresponds with the mean-reversion literature which states that shocks in the short-term interest rate have no permanent effect. The short-term interest rate returns eventually to its unconditional mean. This first specification can be rewritten in a VAR system based on the levels of the three interest rates.

In the second version of the model, the spreads are analyzed together with the first differences in the short-term interest rate. A shock in this latter variable has a permanent effect on the level of the short-term interest rate. Once the shock to Δr is died out, Δr will return to its equilibrium, while the level of r has undergone a permanent shift, *e.g.* in the new equilibrium there are no further changes in r but the cumulative change $\sum_{t=0}^{\tau} \Delta r_t$ is not zero. Hence, this model has a unit root and two cointegrating vectors and is referred to as the *unit root specification*. Other models which allow for a greater number of unit roots are empirically rejected in Table 6.1. The spreads are already stationary so that checking for more unit roots is superfluous. Moreover, those models are less relevant from an economic point of view, and are therefore left out of this analysis.

6.4 VAR analysis

6.4.1 Determining the order

A VAR analysis starts with the determination of the number of lags that have to be included in the system. Here we determine the optimal order p by minimizing the Akaike's information criterion AIC:

$$AIC(p) = \ln \left|\tilde{\Sigma}(p)\right| + \frac{2n^2 p}{T}, \tag{6.19}$$

were $\tilde{\Sigma}(p)$ is the maximum likelihood estimator of the error covariance matrix, n is the number of variables in the system, and T is the number of observations. For the stationary specification of the VAR model we found that the optimal order p equals 2, for the unit root specification the order was found to be $p = 3$.

6.4.2 Granger causality

The first test we apply to study the dynamic interactions between the interest rate variables is the Granger causality test. With this test we survey whether only the short-term interest rate has explanatory power or that fluctuations in the short-term interest rate can be explained by information regarding the long-term interest rate and the mortgage rate. In other words, the direction of causality between variables is investigated. Does the short-term interest rate cause the mortgage rate? Does the mortgage rate cause the short-term interest rate? Or is it the long-term interest rate that causes the mortgage rate? Question like these can be studied with the causality test developed by Granger (1969), hence the term *Granger causality*.

A variable w_t is said to be Granger-caused by a variable z_t if the predictions of w_t can be improved by including the history of z_t in the information set used for the prediction of w_t. More formally, the variable z_t is Granger-causal for the variable w_t if

$$\sigma^2(w_t \mid \mathbf{X}_t, \mathbf{Z}_t) < \sigma^2(w_t \mid \mathbf{X}_t), \tag{6.20}$$

where

$$\mathbf{X}_t = \{x_t, x_{t-1}, \ldots\}, \quad (6.21)$$
$$\mathbf{Z}_t = \{z_t, z_{t-1}, \ldots\}, \quad (6.22)$$
$$\mathbf{Z}_t \notin \mathbf{X}_t, \quad (6.23)$$

and σ^2 represents the conditional variance of w_t.

In a VAR framework testing for Granger causality comes down to a joint test for the estimated coefficients in the VAR system to be equal to zero. That is, to analyze if the first variable in the VAR model Granger-causes the second variable, we have to test the null hypothesis that the parameters corresponding with the first variable in the second OLS regression in the VAR system, are equal to zero. The test statistic is a Wald test with p and $T - pn - 1$ degrees of freedom.

The results of the Granger causality tests are summarized in Table 6.3. Since the level of the short-term interest rate and the first differences of this rate are not included in the same specification of the VAR model, there are no results available for the Granger causality test between these two variables.

TABLE 6.3: GRANGER CAUSALITY TESTS

Jan.72-Dec.95		to			
		r	Δr	$l-r$	$y-r$
	r	•	N.A.	.	.
	Δr	N.A.	◇	.	.
from	$l-r$	.	.	•	◇
	$y-r$	◇	.	?	•

The tables show the results of the Granger causality. A dot indicates that in both VAR-models the row-variable Granger-causes the column-variable at a significance level of 1%, a diamond denotes significance at a 10% level. The question mark denotes that the test results differ between the two VAR-models, and a period means that the row-variable Granger-causes the column-variable in neither VAR model.

Table 6.3 shows that the history of each variable contains information that can help to improve the forecast of its own current value. The mutual relations between the economic variables are less unequivocal. In the stationary specification of the VAR model, the spread between the mortgage rate and the short-term interest rate Granger-causes the spread on long-term government bonds. However, this Granger causality is non-existing in the unit root specification. At a 10% significance level the term structure spread Granger-causes the mortgage spread, while in its turn the spread on mortgages Granger-causes the short-term interest rates. Hence, the spread on long-term government bonds might indirectly Granger-cause the short-term interest rate. Impulse response functions are well-suited to analyze such indirect interactions among variables.

6.4.3 Impulse response functions

An impulse response function examines the effect a shock to one of the endogenous variables has on current and future values of all variables in the system. By studying successive shocks in the three variables we can address the question whether the dynamics of the mortgage rate can correctly be described by a one-factor interest rate model.

TABLE 6.4: OVERVIEW

Shock in	Line Type
r	solid
l	broken
y	dots and dashes

The table shows the line types corresponding with the variables as applied in the impulse response functions and the variance decompositions illustrated in the Figures 6.2 through 6.7.

Figure 6.2 relates to the stationary specification of the VAR model. In this impulse response analysis the first shock occurs in the short-term interest rate. The size of this shock equals the standard deviation of the residuals in the short-term interest rates, which remain unexplained by the VAR model. Note that each graph in Figure 6.2 has its own scale on the vertical axis for visual clarity.

The solid line in the upper diagram of Figure 6.2 reflects the effect a shock in the short-term interest has on the rate itself. The mean-reversion characteristics of the short-term interest rate are obvious. The half-life of a shock is 6 months. After these 6 months the shock fades further until it completely dies out. The responsiveness to the innovation in the short-term interest rate differs substantially between the long-term interest rate and the mortgage rate. The mortgage rate turns out to be more sensitive to shocks in the short-term interest rate than the long-term interest rate. It holds for both interest rates that the instantaneous response is rather small, their impulse response functions reach their maximums after 2 months, while the short-term interest rate peaks one month after the occurrence of the shock. This lagged response is in contrast with the one-factor interest rate models which imply that the long-term interest rate and the mortgage rate should immediately move in the same direction as the short-term interest rate.

Contrary to the single factor assumption, the broken line in the lower diagram of Figure 6.2 shows that an additional shock in the long-term interest rate has an substantial effect on the level of the mortgage rate. Apparently, the long-term interest rate contains information about the mortgage rate dynamics which is not embodied in the short-term interest rate.

An additional innovation in the mortgage rate has an effect on itself but fades quickly. Since this shock occurs after the innovations in both the short and long-term interest rate have been absorbed, it is unrelated to the term structure. This shock in the mortgage rate

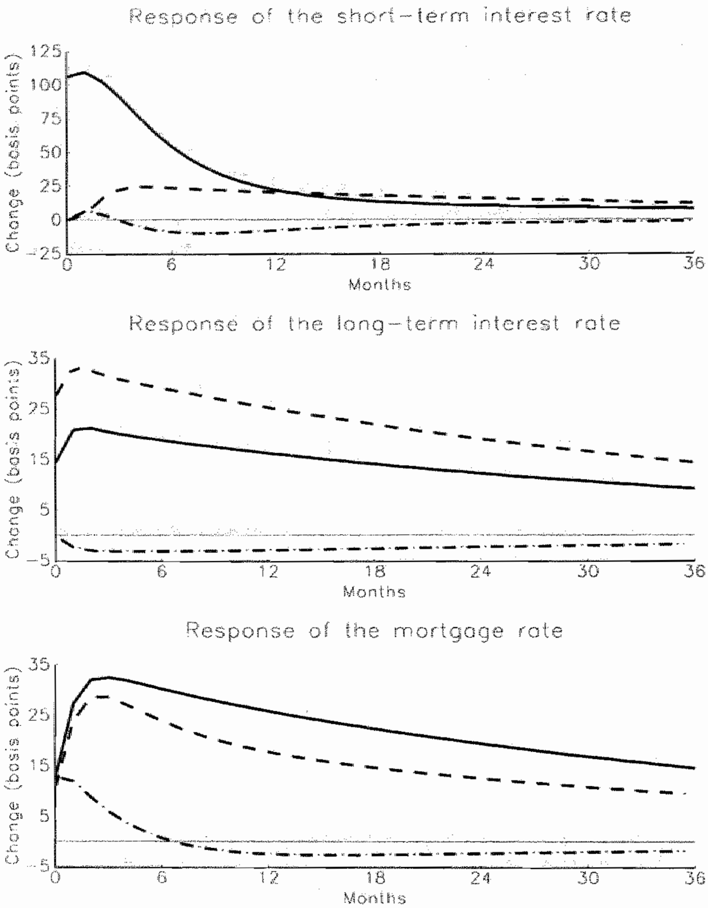


FIGURE 6.2: IMPULSE RESPONSE FUNCTIONS, ORDERING: $r \rightarrow l \rightarrow y$

The solid line in the upper diagram reflects the effect a shock in the short-term interest rate has on the short-term interest rate itself. In the middle diagram, this solid line illustrates the effect a shock in the short-term interest rate has on the long-term interest rate, while in the bottom diagram the effect on the mortgage rate is illustrated. The broken line in the upper diagram shows the response of the short-term interest rate to a subsequent shock in the long-term interest rate. The line which consists of dots and dashes illustrates the response of the short-term interest rate to a shock in the mortgage rate. The lines in the middle and bottom diagrams can be interpreted similarly. This figure is based on the stationary specification of the VAR model.

initially has a small positive effect on the short-term interest rate. However, as soon as one month later, the effect becomes negative and eventually dies out. The long-term interest rate decreases as a consequence of the shock, but the negative effect is very small.

Figure 6.2 shows the shortcomings of a single factor model to model the mortgage rate dynamics. The long-term interest rate turns out to contain additional information regarding mortgage rate fluctuations. The impact of additional innovations in the mortgage rate itself is much smaller. These results can be interpreted as support for a two-factor model.

In Figure 6.2, a shock in the short-term interest rate was orthogonalized first. Often, however, economists argue that mortgage rates are related to long-term interest rates rather than short-term interest rates. To check whether a single factor model based on the long-term interest rate can correctly model the mortgage rate dynamics, we consider a second ordering. In Figure 6.3 the impulse response functions are plotted such that the innovation in the long-term interest rate is analyzed first. A shock in the short-term interest rate is analyzed second, and finally, the effects of an additional shock in the mortgage rate are studied.

The broken line in the upper panel shows that a shock in the long-term interest rate has a substantial instantaneous effect on the short-term interest rate. The long-term interest rate itself increases much less than the short-term interest rate. The mortgage rate reacts to the shock in a similar manner as the long-term interest rate: the effect is at its maximum two months after the shock occurred. Past this point it fades slowly, but the mortgage rate is still more than 18 basis points above its initial level after 36 months.

The solid line in the upper diagram of Figure 6.3 demonstrates that a successive innovation in the short-term interest rate has a serious effect on the short-term interest rate. The effect on the long-term interest rate is rather small and the instantaneous effect on the mortgage rate is also limited. As the lower diagram of Figure 6.3 shows, the mortgage rate reacts with a lag on an additional shock in the short-term interest rate. Finally, in this ordering of the variables, the effects of a shock in the mortgage rate are similar to the effects such a shock had in the previous ordering.

The results suggest that significant improvements can be achieved by using a two-factor model to describe the mortgage rate dynamics.

In Figure 6.4, the impulse response functions are plotted for the unit root specification of the VAR model. A shock in the short-term interest rate has a permanent effect on its level. By construction it also has a permanent effect of the same size on the level of both the long-term interest rate and the mortgage rate. The innovation in the short-term interest rate results therefore in an increase of each of the three interest rates by almost 18 basis points.

The solid lines in the middle and bottom graphs clearly illustrate the lagged response of the long-term interest rate and the mortgage rate on shocks in the short-term interest rate. A one-factor model thus has serious shortcomings in this setting as well. This is also demonstrated by the broken lines in Figure 6.4, which show the responses of an additional

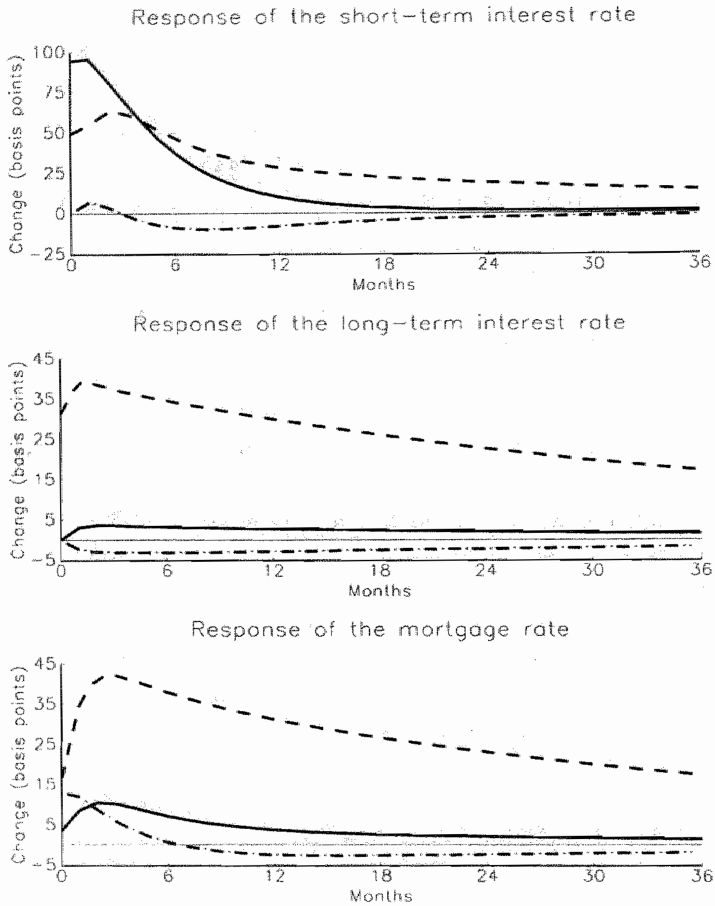


FIGURE 6.3: IMPULSE RESPONSE FUNCTIONS, ORDERING: $l \to r \to y$

For an explanation of the lines see Table 6.4 and Figure 6.2. Note that here the variables are ordered differently. This figure is based on the stationary specification of the VAR model.

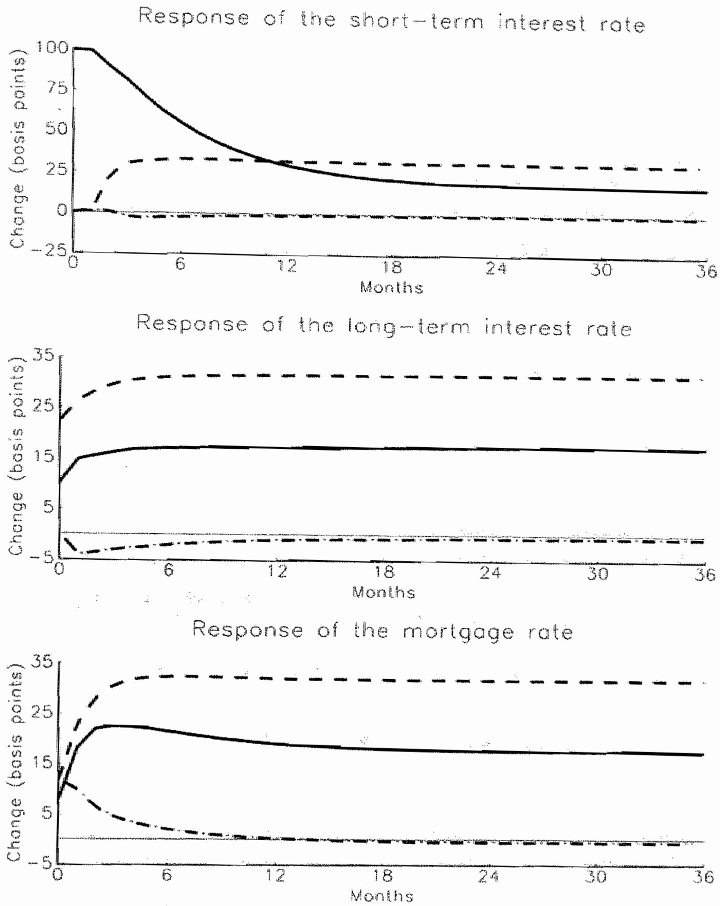


FIGURE 6.4: IMPULSE RESPONSE FUNCTIONS, ORDERING: $r \rightarrow l \rightarrow y$

The solid line in the upper diagram reflects the effect a shock in the first differences of the short-term interest rate has on the level of the short-term interest rate. In the middle diagram, this solid line illustrates the effect such a shock has on the long-term interest rate, while in the bottom diagram the effect on the mortgage rate is illustrated. The broken line in the upper diagram shows the response of the short-term interest rate to a subsequent shock in the long-term interest rate. The line which consists of dots and dashes illustrates the response of the short-term interest rate to a shock in the mortgage rate. The lines in the middle and bottom diagrams can be interpreted similarly. This figure is based on the unit root specification of the VAR model.

shock in the long-term interest rate. This successive shock has a substantial and permanent effect on all three interest rates, thereby stressing the importance of a second factor.

A third shock, this time in the mortgage rate, has only a temporary effect. After one year the effects of this shock almost completely die out for all three interest rates.

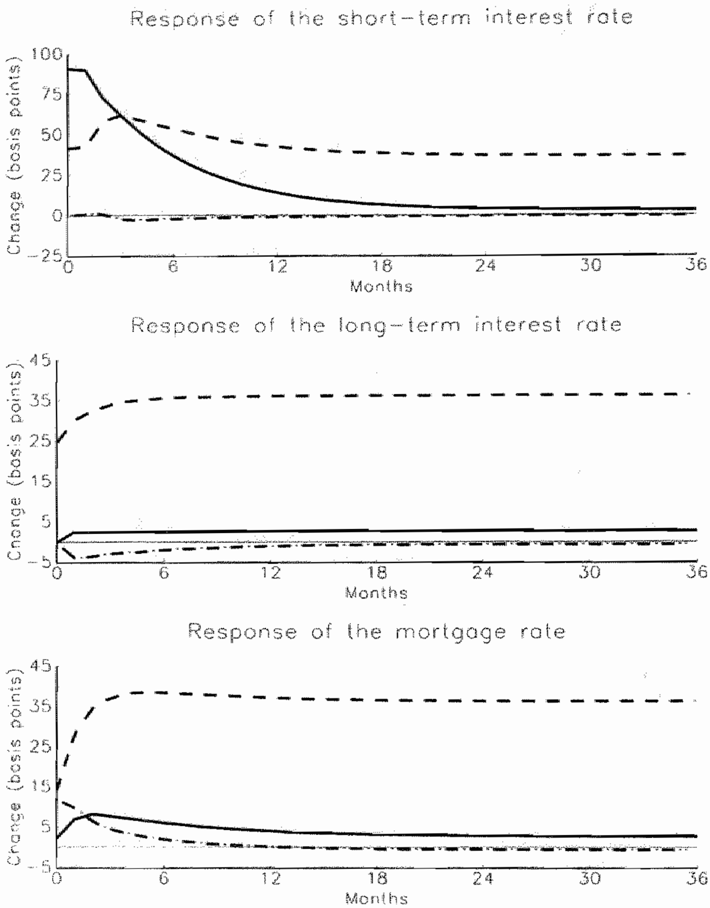


FIGURE 6.5: IMPULSE RESPONSE FUNCTIONS, ORDERING: $l \rightarrow r \rightarrow y$

For an explanation of the lines see Table 6.4 and Figure 6.4. Note that here the variables are ordered differently. This figure is based on the unit root specification of the VAR model.

Changing the order in which shocks occur in the unit root specification of the VAR model results in the impulse response functions plotted in Figure 6.5. Here a shock in the long-term interest rate is orthogonalized first. The broken lines in Figure 6.5 represent the effects this shock has on the endogenous variables. As a results of this innovation the three interest rates increase by 36 basis points. This new level is reached rather quickly by the separate interest rates. The fluctuation in the mortgage rate due to the initial shock in the long-term interest rate is quite small after 6 months.

The long run effect of a successive shock in the short-term interest rate is much smaller. Initially, the short-term interest rate increases substantially. This effect fades fast, however, and after three years the remaining impact is less than 3 basis points. The long run effects of a shock in the mortgage rate are even smaller: -0.6 basis points.

In this specification of the VAR model, a one-factor model with the long-term interest rate as the leading indicator seems to describe the long run dynamics of the mortgage rate rather well. However, as Figure 6.5 illustrates, over the short run the effects of a fluctuating short-term interest rate can not be neglected.

To conclude, even though one-factor interest rate models allow mathematical derivations of deterministic equations to price interest rate derivatives, the results in this subsection indicate that a single factor does not correctly describe the dynamics of the interest rate term structure and that more factors should be included.

6.4.4 Variance decompositions

Variance decompositions isolate the relative importance of random shocks to the forecast errors of the variables. Given Equation 6.18, the error variance of the s periods-ahead forecast of a variable can be decomposed into the components accounted for by shocks in the individual variables. In other words, the forecast uncertainty is assigned to the separate variables in the system. The single factor assumption prescribes that the forecast errors of the interest rates included in our VAR system depend solely on the short-term interest rate. Whether this is true or not is studied in this subsection.

Similar to the impulse response functions, the variance decomposition analysis depends heavily on the ordering of the variables. The orderings worked out in Figures 6.6 and 6.7 are summarized in Table 6.5.

TABLE 6.5: ORDERING

Graphs	Ordering
Left columns	$r \longrightarrow l \longrightarrow y$
Right columns	$l \longrightarrow r \longrightarrow y$

The table shows the ordering of the variables as applied in the variance decompositions illustrated in the Figures 6.6 and 6.7.

The variance decompositions summarized in Figure 6.6 are based on the stationary specification of the VAR model. The graphs on the left-hand side of this figure illustrate the variance decompositions when a shock in the short-term interest rate is followed by

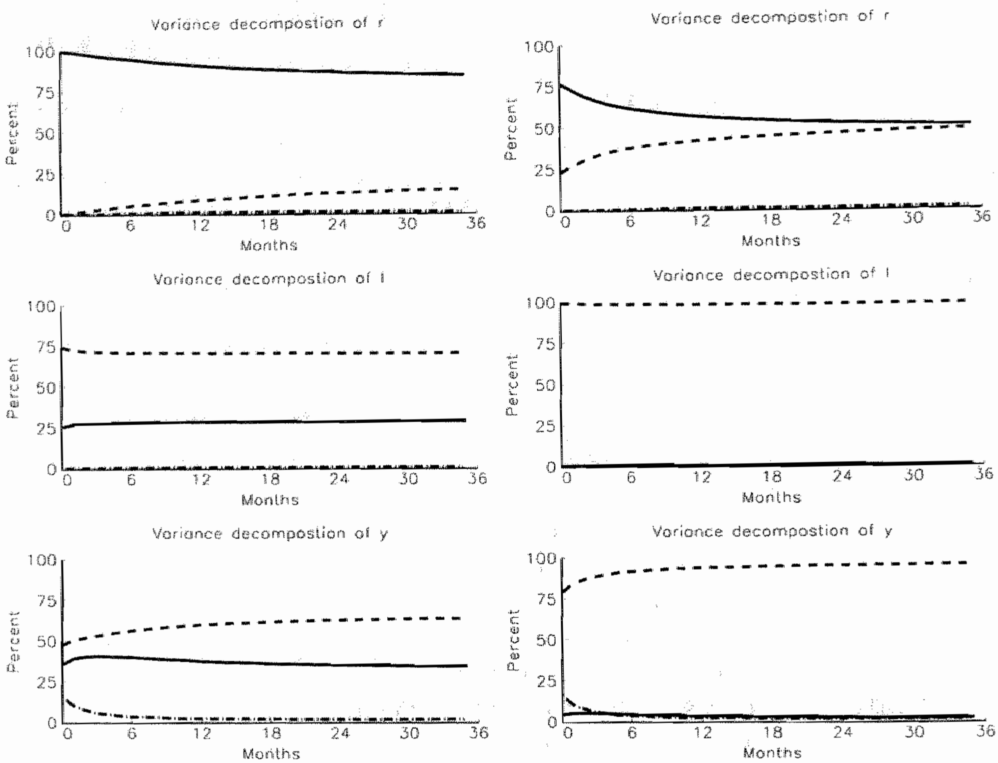


FIGURE 6.6: VARIANCE DECOMPOSITION, STATIONARY PROCESS

The graphs on the left-hand side of this figure illustrate the variance decompositions when a shock in the short-term interest rate is followed by an innovation in the long-term interest rate, and subsequently by a shock in the mortgage rate. The right diagrams show the variance decompositions when a shock in the long-term interest rate is followed by an innovation in the short-term interest rate, and subsequently by a shock in the mortgage rate. In keeping with Table 6.4, the solid lines represent the impact of the short-term interest rate, the broken lines refer to the long-term interest rate and the lines made up of dots and dashes reflect the relative importance of innovations in the mortgage rate.

an innovation in the long-term interest rate, and subsequently by a shock in the mortgage rate. In this setting, 84% of the variance of the 36 periods-ahead forecast of the short-term interest rate is caused by fluctuations in the short-term interest rate itself. Fluctuations in the long-term interest rate explain 15% of the variance, with the remaining part being due to fluctuating mortgage rates. As the middle diagram shows, only 29% of the variance of the long-term interest rate forecast is explained by fluctuations in the short-term interest rate. Even when the long-term interest rate is orthogonalized second, it still accounts for more than 70% of its own variance. Looking at the variance decomposition of the mortgage rate we see that 34% of the variance can be explained by fluctuations in the short-term interest rate. The majority of the variance in the mortgage rate is caused by varying long-term interest rates. If the long-term interest rate is orthogonalized first, 96% of the variance of the mortgage rate forecast of 36 periods-ahead is due to fluctuating long-term interest rates. This is illustrated in the bottom right-hand diagram of Figure 6.6.

The six diagrams of Figure 6.6 display that a one-factor model based on the short-term interest rate falls short in describing the mortgage rate dynamics. The long-term interest rate turns out to contain information about the dynamics of the mortgage rate which is not embodied in the short-term interest rate.

Figure 6.6 is based on the stationary specification of the VAR model. The variance decompositions of the unit root specification, with Δr replacing the level of the short-term interest rate r, are plotted in Figure 6.7. In this unit root specification, the influence of shocks in the long-term interest rate is larger than before. This time 35% of the variance of the 36 months-ahead forecast of the short-term interest rate can be explained by innovations in the long-term interest rate. For the variance in the long-term interest rate and mortgage rate this is 77 and 73%, respectively. As the lower two panels on the left-hand side in Figure 6.7 show, the explanatory power of the short-term interest rate is much less than this.

This result becomes more obvious by changing the order in which the shocks occur. This is demonstrated by the plots on the right-hand side of Figure 6.7, where the long-term interest rate is orthogonalized first. After 36 months 65 percent of the variance in the short-term interest rate can be explained by a shock in the long-term rate. And this percentage continues to increase even after that time. With regard to the long-term interest rate and mortgage rate, we can conclude that in the long run, almost 100% of the variance of these two variables can be explained by the long-term interest rate.

Figures 6.6 and 6.7 clearly indicate that one-factor interest rate models have serious shortcomings when the short-term interest rate is the leading indicator. When modelling the volatility of the mortgage rate, a single factor model based on the long-term interest rate seems to be capable of explaining most of the long run variance. However, such a one-factor model fails to describe the short run volatility of the short-term interest rate correctly.

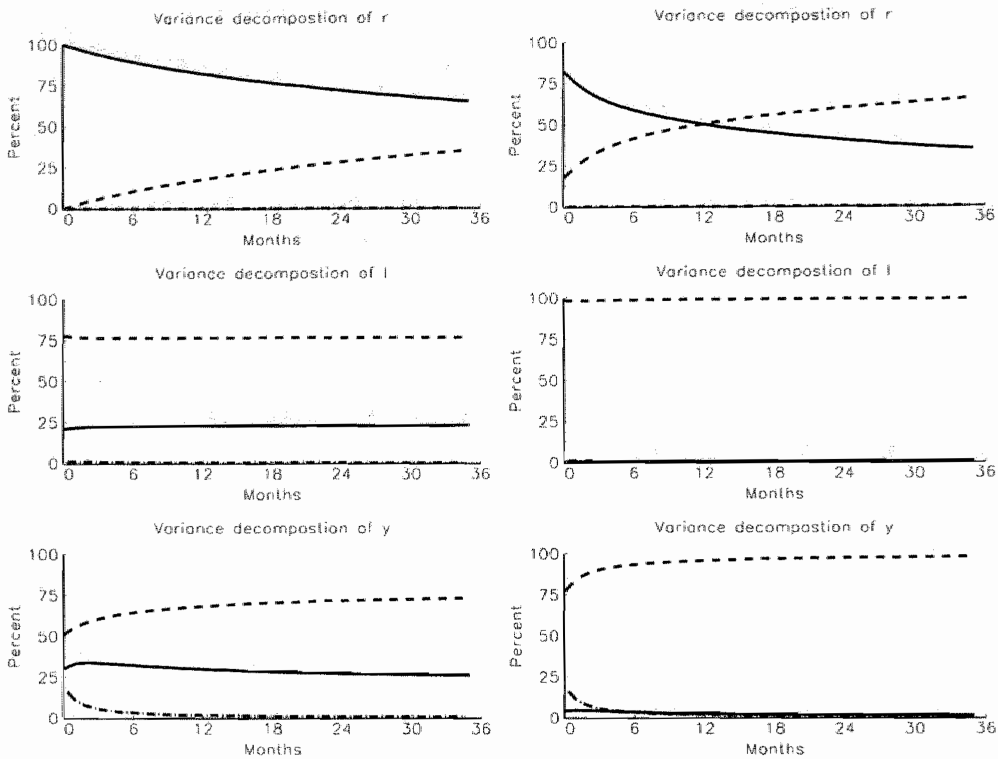


FIGURE 6.7: VARIANCE DECOMPOSITION, UNIT ROOT PROCESS

The graphs on the left-hand side of this figure illustrate the variance decompositions when a shock in the first difference of the short-term interest rate is followed by an innovation in the long-term interest rate, and subsequently by a shock in the mortgage rate. The right diagrams show the variance decompositions when a shock in the long-term interest rate is followed by an innovation in the first differences of the short-term interest rate, and subsequently by a shock in the mortgage rate. In keeping with Table 6.4, the solid lines represent the impact of the short-term interest rate, the broken lines refer to the long-term interest rate and the lines made up of dots and dashes reflect the relative importance of innovations in the mortgage rate.

6.5 Conclusion

Many mortgage pricing models are based on the assumption that mortgage rate dynamics can be described by a single factor interest rate model. This chapter questions whether or not this assumption holds. For this, we analyze the empirical relation between the short-term interest rate, the long-term interest rate, and the mortgage rate in the Netherlands. Vector AutoRegressive techniques are utilized to study the dynamic interactions between these variables.

The results of Granger causality tests, impulse response functions and variance decompositions show that single factor interest rate models have serious shortcomings in describing the dynamics of the mortgage rate. For example, the Granger causality tests reveal that both the short-term and the long-term interest rate contain information about the mortgage rate. The impulse response functions demonstrate that innovations in the long-term interest rate have an additional effect on the level of the mortgage rate. Also, variance decompositions show that fluctuations in the long-term interest rate explain most of the variance of the mortgage rate. A one-factor model based on the long-term interest rate is therefore better capable of modelling mortgage rates than a model solely based on short-term interest rates. However, the short-run mortgage rate dynamics are better described by this latter variable.

Consequently, a one-factor interest rate model is not sufficient in accurately describing mortgage rate dynamics. Having highlighted the need for additional factors, the next chapter develops a multi-factor model in order to price mortgages.

Chapter 7

Multi-factor interest rate models and the valuation of Dutch mortgages

7.1 Introduction

No closed-form solutions are available to calculate the exact values of complex fixed-income securities like mortgages. For this we have to turn to numerical solution methods such as binomial trees, finite difference methods, and Monte Carlo simulations. The first two procedures have serious computational drawbacks when several underlying variables are used or when the payoff of the security depends on the history of the underlying variable. In this case the number of branches in the interest rate tree expands rapidly. For large numbers of time steps, the tree becomes too complex to be an efficient numerical procedure to solve the valuation problem.

The results of Chapter 6 indicate that empirical mortgage rate dynamics can be modelled more accurately when more than one factor is utilized. In keeping with this, a multi-factor mortgage pricing model is developed. The short-term interest rate, the long-term interest rate and the mortgage rate are included as the state variables in this multi-factor model. The dynamic interactions between these variables are described by the VAR models developed in the previous chapter. The resulting VAR-parameters are utilized to simulate short-term, long-term and mortgage interest rates which are the input of the valuation procedure. As in Chapter 6, two alternative VAR specifications are utilized: with and without a unit root for interest rates.

The advantage of a simulation procedure for mortgage pricing is that it can cope with complicated stochastic interest rate environments while simultaneously allowing detailed prepayment restrictions. Its largest disadvantage is that its forward-looking characteristics prevent it from being completely compatible with the dynamic programming solution.

Several studies apply simulation techniques to value Mortgage-Backed Securities. For example, Zenios (1993), Cagan, Carriero and Zenios (1993) and Paskov and Traub (1995) discuss the valuation of MBS by means of Monte Carlo methods. Zenios (1993) uses empirical prepayment data provided by Bear Stearns to generate cash flows for each interest rate scenario. Cagan, Carriero and Zenios (1993) assume that the Mortgage-Backed Security is fairly priced by the market and focus on the calculation of the option-adjusted spread. However, rather than elaborating on the economic interpretation of their results, Cagan, Carriero and Zenios stress the importance of speeding up the simulation procedure. The same holds true for Paskov and Traub who also use an empirically determined prepayment rule.

Schwartz and Torous (1989) employ Monte Carlo simulation methods to generate pseudo time series of the instantaneous risk-free interest rate and the long-term Treasury rate. Similar to Brennan and Schwartz (1985), Schwartz and Torous (1989) assume that all information regarding the term structure of interest rates can be summarized by a short-term and long-term interest rate. This two-factor interest rate model is extended with a proportional-hazards model, as in Green and Shoven (1986), to estimate the influence of various explanatory variables on the prepayment activity. Hence, the prepayment function in the Schwartz and Torous (1989) model is completely empirically determined. This is in contrast to McConnell and Singh (1993), who start with an optimal value-minimizing call condition. The outcome of this first step is a series of critical boundaries similar to the ones derived in Chapter 5. In the second step, McConnell and Singh use Monte Carlo simulation techniques to generate interest rate paths. Prepayment is triggered when the interest rate path hits the critical boundary. This second step is necessary because the Collateralized Mortgage Obligations (CMO) valued by McConnell and Singh have a hierachical structure of payments to the various tranches which requires knowledge of prior mortgage prepayments.

The contracts studied in this chapter include typical Dutch features that are not captured by the models of Schwartz and Torous (1989) and McConnell and Singh (1993), *i.e.*, the annual prepayment restriction and the minimum interest rate guarantee included in many quotation offers are considered. We include transaction costs and concentrate on interest rate driven prepayments.

Similar to the CMOs valued by McConnell and Singh, Dutch mortgages with annual prepayment restrictions are path-dependent in the sense that past prepayments influence future cash flow patterns. Since a multi-factor process is used to model the interest rate dynamics we cannot derive the critical boundary by backward recursion as in McConnell and Singh. Instead we assume that the mortgage is refinanced as soon as this reduces the future cost for the borrower. This prepayment rule is compared with the optimal rule in Chapter 5.

The chapter is organized as follows. Section 7.2 describes the mortgage contracts. Section 7.3 discusses the overall design of the valuation procedure. The applied interest rate risk benchmarks are summarized in Section 7.4. The valuation results follow in Section 7.5. Section 7.6 studies the sensitivity of the valuation results for the considered period. In Section 7.7 the results are compared with the results of the one-factor valuation algorithm developed in Chapter 5. And Section 7.8 concludes.

7.2 The mortgage contract

The mortgage contracts considered in this chapter are very similar to the ones analyzed in Chapter 5. The valuation results from Chapter 5 are therefore used as a benchmark. The mortgage is an annuity which pays a constant monthly cash flow consisting of both interest and redemption. The maturity of the contract is thirty years with the contract rate being fixed for a five year period. After these five years, the contract rate is freely reset so that the interest rate risk does not extend to periods after the adjustment date.

In Chapter 5 fully callable contracts were studied. However, most Dutch mortgage contracts are limited in this. Within a calendar year usually only 10 to 20% of the initial loan can be called without heavy penalties. These restrictions are included in the contracts studied in this chapter and compared with noncallable and freely callable characteristics, whereby we assume that all contracts start on January 1^{st}.

Preceding a mortgage contract, a Dutch bank normally offers a quotation which is valid for a few months. During these months, the client can consider the offer, while at the same time he is guaranteed the lowest contract rate realized on the mortgage market in that period. This offer is a free option for the client; there are no costs involved if he decides not to accept this offer. On the other hand, the writer of this option faces additional interest rate risk for which he would like to be reimbursed. The quotation offer is normally effective for three months. Occasionally, this period is as large as six months.

7.3 Valuation procedure

The primary components of the valuation model are illustrated in Figure 7.1. The valuation procedure takes as input observed interest rates. The relations between those interest rates are described by Vector AutoRegressive (VAR) models for the short-term interest rate, a long-term interest rate and a mortgage rate. The resulting VAR parameters are used to generate interest rate paths. Along these paths the cash flow patterns are determined by taking interest rate driven prepayments in consideration. These cash flows are discounted with the simulated short-term interest rates adjusted for the market price of interest rate risk. The mortgage value is found by repeating this procedure many times and averaging the discounted cash flows.

7.3.1 Interest rates simulation

In Chapter 6, two specifications of a VAR model are utilized to estimate the dynamic interactions between the short-term interest rate, the long-term interest rate and the mortgage rate in the Netherlands. In the first specification a stationary process is analyzed, and the second one includes a unit root. The weights of shocks in a stationary process decline geometrically with time such that they have no permanent effects, and the interest rates eventually return to their unconditional means. In the second specification of the VAR model, shocks have permanent effects on the interest rates levels. Both specifications are

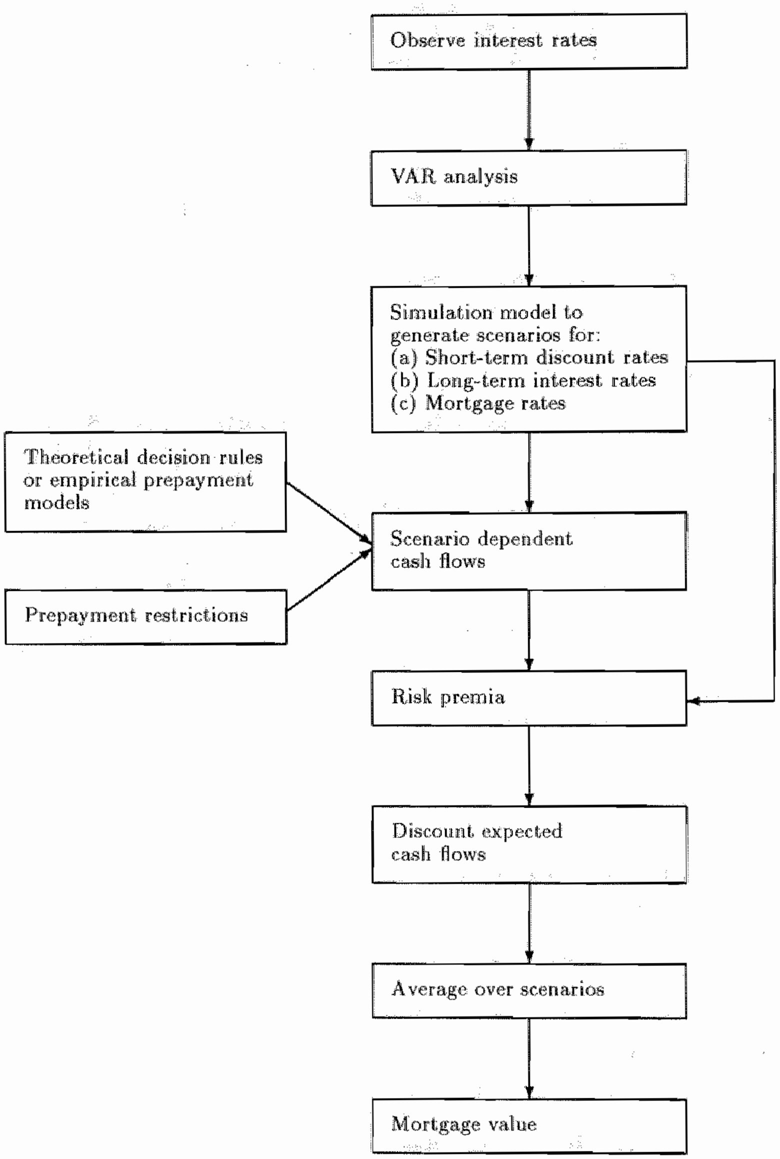


FIGURE 7.1: SIMULATION PROCEDURE

discussed in detail in the previous chapter where the parameters are estimated for both versions of the VAR model.

A Monte Carlo simulation is carried out to generate various interest rate paths. The variability of the results is reduced by using the antithetic variable technique.[1] The stochastic terms in these simulations are generated by the following bootstrapping procedure. Let ε_t be a (3×1) vector which contains the residuals from the VAR model at time t. By randomizing with replacement over the dimension of time, a vector $\tilde{t}$ is created with a length n which equals the length of the simulated interest rate paths. Each element in $\tilde{t}$ corresponds with a month between January 1972 and December 1995. We then create a pseudo-history by selecting the cross section stochastic error terms that correspond to the bootstrapped dates. This way an $(3 \times n)$ matrix $E_{\tilde{t}}$ is created which contains the stochastic error terms for the n-periods that the interest rates are simulated. By bootstrapping the errors from the VAR model in this way, we preserve the distributional characteristics of the correlated interest rate innovations.

Once the parameters are known and the residuals are drawn, only the initial interest rates have to be chosen before the simulations can be run. For the short-term interest rate 4, 8 and 12% were taken. The corresponding long-term interest rate and mortgage rate are found by increasing these short-term interest rates with the average spread observed in the sample period whenever the short-term interest rate was equal to 4, 8 or 12%, respectively. Consequently, these spreads differ for the different levels of the initial spot rates. Finally, we assume that the starting lagged values of the interest rate variables are the same as those at time $t = 0$.

Table 7.1 shows the average rates and standard deviations of the short-term, long-term and mortgage interest rates at the end of the 60 months' fixed-rate period. These results are based on 30,000 simulation runs.

The differences between a stationary and a unit root process are obvious. Even though the rates are not yet completely converged, Table 7.1 clearly points out the mean-reversion in the stationary process. If we lengthen the period, the short-term interest rate converges to 6.81%, the long-term interest rate becomes 7.89%, and the unconditional mean of the mortgage rate is 8.84%. These results are independent of the initial rate, which is in sharp contrast with the results of the unit root process. Here the sixty months ahead interest rates depend on the starting value of the interest rate. Table 7.1 also shows that for the unit root process, the average generated spreads between the mortgage rates, the long-term interest rates and the short-term interest rates are independent of the initial spot rate.

To preclude negative interest rates, the paths were truncated when necessary. This causes the standard deviations to be relatively low at an initial spot rate of 4%. Especially when there is no upward drift at such a low interest rate this has a substantial impact on the standard deviations and explains why the volatility at 4% is lower than at higher rates.

[1] In essence the antithetic variable technique comes down to simulating two paths at the same time by using both the positive and negative value of the drawing. That is, if ε is included in the first path, $-\varepsilon$ is used in the second. See Hull (1997) for a more detailed discussion.

TABLE 7.1: DESCRIPTIVE STATISTICS OF THE INTEREST RATES

VAR specification	stationary			unit root		
Initial spot rate	4%	8%	12%	4%	8%	12%
	After 60 months					
Short-term interest rate	6.39	7.09	7.64	4.76	7.81	10.65
Standard deviation	2.25	2.36	2.43	2.41	3.28	3.60
Long-term interest rate	7.38	8.22	8.88	5.94	8.98	11.82
Standard deviation	1.44	1.50	1.51	2.29	2.76	2.86
Mortgage rate	8.32	9.18	9.85	6.90	9.95	12.78
Standard deviation	1.46	1.52	1.53	2.27	2.75	2.86

The table shows the average rates and standard deviations of the short-term, long-term and mortgage interest rates at the end of the 60 months' fixed-rate period. The results are derived by 30,000 simulation runs based on VAR analyses of Dutch interest rates between 1972 and 1995.

In addition to Table 7.1, Figure 7.2 displays the average interest rate paths generated by the stationary specification of the VAR model. These paths are based on the same 30,000 simulation runs. The upper diagrams clearly illustrate the mean-reversion in this process. Independent of the initial rates, the interest rates converge to the same levels. These diagrams also show the differences between the initial spreads at different starting rates. For example, at an initial short-term interest rate of 8 percent the spread between the long-term interest rate and the mortgage rate is only 20 basis points, while at 12 percent the initial spread between the short-term and long-term interest rate is very small. The lower diagrams yield a close up look of the mortgage rates. These rates determine the opportunity costs for the borrower who considers refinancing. Therefore, they influence the prepayment behavior which on its turn affects the valuation of the mortgage. In Section 7.5, where we discuss the results, we will frequently refer to this figure.

7.3.2 Cash flows and prepayment behavior

Annuity-mortgage contracts pay a constant monthly cash flow which consists of both interest and redemption payments. The periodical cash flows, M_0, are a function of the contract rate, the size of the principal and the time to maturity:

$$M_0 = U_0 \frac{(1-v)}{v(1-v^T)}, \tag{7.1}$$

where U_0 is the principal, T is the time to maturity and $v = \frac{1}{(1+y(0))}$, with $y(0)$ being the contract rate determined at time $t = 0$.[2]

Besides these factors, the prepayment behavior of the borrower influences the cash

[2] Consistent with Chapter 5, the time demarcations for the mortgage rate y and the short-term interest rate r are parenthesized. For all other variables the time demarcation remains subscripted.

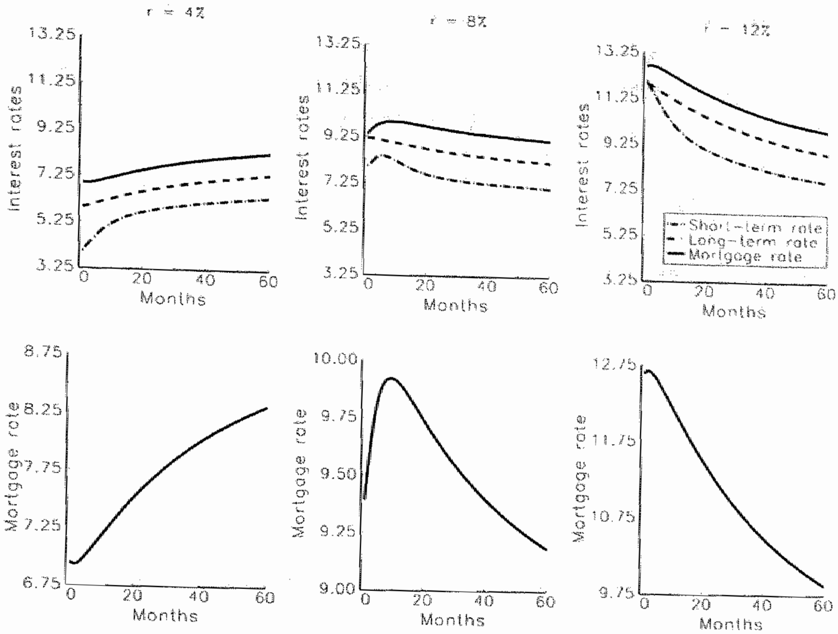


FIGURE 7.2: AVERAGE INTEREST RATE PATHS

The upper diagrams of this figure show the average interest rate paths generated by the stationary VAR model. The lower diagrams yield a close up look at the mortgage rates. The r value on top of each column represents the short-term interest rate at which the simulations started. The results are derived by 30,000 simulation runs based on a VAR analysis of Dutch interest rates between 1972 and 1995.

flow pattern. With prepayments playing such an important role, it is necessary to model them accurately. Depending on the scope of the study two approaches are possible. First, prepayment behavior can be described by a formal econometrical model estimated by using historical data. This approach is preferable if the goal of the research is in modelling the observed economic behavior. This is done in Chapter 9 which studies the Dutch prepayment experience of mortgages in the early nineties. That study suggests ways in which the prepayment behavior should be modelled more formally, but is not yet suited for the applications in this chapter. For that, more accurate and comprehensive data must be collected, not only regarding prepayment rates themselves but also about the many variables that influence them. Since both the quantity and quality of the available data on the Dutch mortgage market are not yet sufficient, the first approach is ruled out.

If the purpose of the study is to analyze the interest rate risks faced by the mortgagee, a theoretical prepayment decision rule is preferable. In this chapter the same rule will be used as in Chapter 5, recognizing that in strict sense this prepayment rule is not optimal.

In this chapter we assume that the mortgage will be prepaid as soon as this reduces the future costs for the borrower. Therefore we calculate each month the monthly payment M_t^* as if a new contract starts with a maturity equal to the remaining time to maturity of the existing contract. The new contract rate is equal to that month's simulated mortgage rate. The principal of this new loan is equal to the outstanding debt multiplied by $(1+c)$, where c are the up-front costs a borrower has to pay to replace the old contract by a new one. If the resulting monthly payment M_t^* is less than the original M_0, the borrower considers prepaying the contract. If he replaces the existing contract with a new one he will save each month, until the reset date, an amount equal to $(M_0 - M_t^*)$. These monthly savings can be deposited on a bank account on which he earns interest. The exact payment of interest, however, depends on future interest rates which are unknown at time t when the prepayment decision must be made. However, the term structure of interest rates can be approximated by interpolating between the short-term interest rate and the long-term interest rate, which are both generated by the simulation procedure. This approximation is used to calculate the expected value of S_τ, which is the amount the mortgagor will have at his disposal on the bank account on the reset date τ if he decides to refinance the contract at time t. This money can be used to reduce the outstanding debt on that day. The borrower is indifferent between refinancing the contract and holding on to it if S_τ is large enough to offset the potential higher debt of the new loan on the reset date.[3] We assume that the mortgagor prepays the contract as soon as:

$$\tilde{U}_\tau - S_\tau < U_\tau, \tag{7.2}$$

where U_τ is the outstanding debt on the reset date τ if the contract is *not* prepaid at time t, while $\tilde{U}_\tau$ reflects what the debt will be at time τ if the contract is prepaid at time t.

Throughout this chapter we assume that the mortgage is prepaid as soon as Inequality (7.2) holds. We exclude exogenous prepayments which are unrelated to interest rates. Nor are prepayments considered which exceed the annual penalty-free restrictions. Consequently, when the borrower decides to (partially) prepay the mortgage he only faces the up-front fees c of starting a new contract. Even though the borrower has to pay these up-front costs, the lender will not receive them since they refer to setting up a new contract and are therefore received by the new issuing party. Initially, these costs are assumed to be equal to one percent of the principal. Later alternative costs are also considered.

7.3.3 Pricing

The pricing model accepts as input the simulated short-term interest rates and the cash flows as determined by the contract and the prepayment behavior model. The present value of these cash flows are calculated by using risk-adjusted discount rates. For this we adjust the unconditional mean of the short-term interest rate such that it includes the

[3] See Section 5.2.4 for a detailed discussion on this prepayment rule.

market price of risk. The alternative VAR specifications demand for different approaches in this. If the short-term interest rate follows a stationary process, the infinite horizon yield converges to R_∞, independently of the initial spot rate. Setting R_∞ at a particular value, say $R_\infty = 8\%$, we can solve for the market price of risk. The same approach was used in Chapter 5.

For the unit root process this approach does not work, since now the short-term interest rates do not converge to one level. To derive an appropriate market price of risk measure, we assume that the price of an n -period zero-coupon bond is equal to its value at time n discounted by the short-term interest rates plus a risk premium over the intermediate periods. In terms of yields this reads:[4]

$$\frac{1}{(1+R_{n,t})^n} = \prod_{s=0}^{n-1} \frac{1}{(1+r_{t+s}+\lambda)^n} \tag{7.3}$$

where $R_{n,t}$ is the yield to maturity on an n-period zero-coupon bond at time t, r_t is the short-term interest rate which holds for one period, and λ is the risk premium. Moving to natural logarithms, Equation (7.3) can be approximated by:

$$n \ln(1+R_{n,t}) \approx \sum_{s=0}^{n-1}(r_{t+s}+\lambda), \tag{7.4}$$

such that

$$R_{n,t} \approx \frac{1}{n}\sum_{s=0}^{n-1}(r_{t+s}+\lambda). \tag{7.5}$$

From which follows:

$$\begin{aligned}
R_{n,t} - r_t &\approx \frac{1}{n}\sum_{s=0}^{n-1}(r_{t+s} - r_t + \lambda), && (7.6)\\
E[R_{n,t} - r_t] &\approx \frac{1}{n}\sum_{s=0}^{n-1}(E[r_{t+s} - r_t] + \lambda, && (7.7)\\
E[R_{n,t} - r_t] &\approx \lambda. && (7.8)
\end{aligned}$$

From Equation (7.8) follows that the average spread between the n-period yield and the short-term interest rate is an appropriate measure for the risk premium. For the unit root process we therefore use the average spread between the short-term and long-term interest rate observed during the sample period to adjust the discount rate such that it includes the market price of risk.

[4] See Campbell, Lo and MacKinlay (1997) for a detailed discussion on this.

7.4 Interest rate risk

The price/interest rate curve of a mortgage shows how sensitive the mortgage value is to interest rate changes. A linear approximation, such as duration, for this interest rate sensitivity results in a substantial error when callable contracts are considered.[5] Therefore, a more direct measure of interest rate risk has been introduced: the *effective* duration Δ, which is defined as the semi-elasticity of the mortgage with respect to the short-term interest rate:[6]

$$\Delta(r(0), y(0)) = -\frac{\partial \ln V(r(0), y(0))}{\partial r(0)} \tag{7.9}$$

where V represents the value of a mortgage contract whose contract rate equals $y(0)$ and $r(0)$ is the initial short-term interest rate. In this chapter a numerical approximation for Δ is used:

$$\Delta(r(0), y(0)) = -\frac{V(r(0) + h, y(0)) - V(r(0) - h, y(0))}{[(r(0) + h) - (r(0) - h)]\, V(r(0), y(0))} \times 100. \tag{7.10}$$

The small deviation of the initial short-term interest rate, h, is assumed to be one basis point. As discussed in the previous chapter, such a shock in the short-term interest rate has an instantaneous effect on both the long-term interest rate and mortgage rate. The size of these effects follow from the Choleski decomposition of the covariance matrix resulting from the VAR analysis. As a consequence of these shocks, the starting values of the short-term, long-term and mortgage interest rate change so that new simulated interest rate paths have to be generated. To determine the stochastic error terms in this, the same matrix $E_{\bar{t}}$ as in Section 7.3.1 is utilized. From the resulting alternative interest rate paths, only the short-term interest rate paths are used. For the mortgage rate, which determines the prepayment behavior and thus the cash flow pattern, the original rates, starting at $y(0)$, are used. This way, we compute the partial derivative and are assured that only the effects of changing discount rates are considered in the interest rate sensitivity analysis. In other words, there will be no discrete changes due to different prepayment behavior.

Acknowledging the drawbacks of duration as a measure of interest rate risk, the results in Section 7.5 present this benchmark as it shows the weighted time to maturity, which is important for the funding of the mortgage.

7.5 Valuation results

This section presents the valuation results based on alternative VAR specifications, initial short-term interest rates and penalty-free prepayment restrictions. Unless stated otherwise,

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[5] See DeRosa, Goodman and Zazzarino (1993) and Haensly, Springer and Waller (1993) for a discussion on this topic.

[6] See Fabozzi and Modigliani, Chapter 13 (1992).

the simulated interest rate paths are based on VAR analyses which took interest rates between January 1972 and December 1995 as input.

The impact of annual prepayment restrictions

Table 7.2 presents the computed values at origination of a noncallable contract and contracts which are limited or fully callable. A quotation offer guarantee is not issued with these contracts, and the up-front costs are set to 1% of the loan. The applied spread between the mortgage rate and the short-term interest rate is based on historical observations in the Netherlands. The mortgage yield is hereby equal to the average yield on annuity-mortgages with a maturity of 30 years and a fixed contract rate period of 5 years. Thereby, no distinction is made between alternative prepayment restrictions. Historically only 10% of the initial loan could be called within a calendar year without penalty. Due to the increased competition this limitation is gradually widened and nowadays there are many contracts which allow the annual prepayments to be as large as 20% of the initial loan. Despite the broadening of prepayment possibilities, the average spread barely increased, as illustrated in Figure 5.3. This reflects the general impression that, due to the annual limitations, prepayment risk is less important in the Netherlands than in the United States where prepayment restrictions are virtually unknown. However, Table 7.2 shows that this is a misunderstanding as the value of a 10% penalty-free prepayment option equals one quarter of the value of a prepayment option without any limitations. The value of the 20% penalty-free prepayment option is even equal to half the value of the 100% penalty-free prepayment option!

The effective duration measures Δ in Table 7.2 show that the interest rate risk faced by the mortgagee decreases with increasing prepayment freedom for the borrower. This side-effect of loosening prepayment restrictions is caused by the increased prepayment probability during early stages of the contract. This phenomenon shortens the average life of a mortgage, as reflected by the duration-pattern in Table 7.2, and therefore reduces the probability of a large change in the contract value.

The effect of mean-reversion on prepayment behavior

The mean-reversion of the stationary process pulls the interest rates towards their unconditional means. In the unit root process, the interest rates have a higher volatility which explains why the option values at a 4 and 8% short-term interest rate level are higher when the unit root process is utilized. At higher interest rates, the downward drift included in the stationary process overpowers the volatility characteristics and results in relative high option values.

What holds true for the option value is also valid for the duration: higher volatility reduces the duration at low rates, while the downward drift shortens the duration at high rates. One duration figure in particular stands out. When the stationary VAR model is utilized and all prepayment restrictions are released, an initial short-term interest rate of 8

TABLE 7.2: MORTGAGE VALUES: SAMPLE PERIOD JAN. 1972 - DEC. 1995

VAR specification	stationary			unit root		
Spot rate	4%	8%	12%	4%	8%	12%
	0% penalty-free prepayment					
Annuity value	100.27	102.44	108.98	104.65	101.78	102.61
Duration annuity	50.34	48.53	46.47	50.66	48.41	45.79
Δ annuity	0.63	0.61	0.58	0.63	0.60	0.57
	10% penalty-free prepayment					
Mortgage value	100.15	101.88	106.39	103.65	101.07	101.65
Duration mortgage	46.19	44.34	37.16	44.41	44.23	40.48
Δ mortgage	0.61	0.59	0.54	0.59	0.58	0.54
Option value	0.12	0.56	2.59	1.00	0.69	0.96
	20% penalty-free prepayment					
Mortgage value	100.04	101.33	103.81	102.63	100.37	100.69
Duration mortgage	42.11	40.18	27.37	37.89	40.06	34.94
Δ mortgage	0.58	0.57	0.50	0.56	0.56	0.51
Option value	0.23	1.10	5.17	2.03	1.41	1.93
	100% penalty-free prepayment					
Mortgage value	99.69	99.94	100.13	100.55	99.00	99.04
Duration mortgage	24.58	27.01	9.52	19.31	27.34	20.35
Δ mortgage	0.44	0.49	0.33	0.39	0.46	0.39
Option value	0.58	2.50	8.86	4.10	2.77	3.60

The table shows valuation results based on alternative VAR specifications, initial short-term interest rates, and penalty-free prepayment conditions. Duration is expressed in months. The *effective* duration Δ is defined as the semi-elasticity of the mortgage with respect to the short-term interest rate. A quotation offer guarantee is not included in the mortgage conract. Utilized interest rates: Jan. 1972 - Dec. 1995, up-front fee: 1% of the loan.

percent leads to a relative high duration of 27.01 months. The explanation for this can be found in the lower-middle diagram of Figure 7.2. This plot displays that, in this starting situation, the mortgage rate initially tends to rise. This reduces the prepayment likelihood and results in a relative high duration measure.

TABLE 7.3: SPREADS

VAR specification	stationary			unit root		
Initial spot rate	4%	8%	12%	4%	8%	12%
Initial spread	295	140	65	295	140	65
Spread after 60 months	193	209	221	214	214	213
Difference	-102	+69	+156	-81	+74	+148

The table shows the spreads (in basis points) between the mortgage rate and the short-term interest rate at times $t = 0$ and $t = 60$, as well as the difference between the spreads at both periods. The initial spreads are based on the average spreads observed in the research period when the short-term interest rate was equal to 4, 8 and 12 percent. The spreads at time $t = 60$ are derived by 30,000 simulation runs based on VAR analyses of Dutch interest rates between 1972 and 1995.

In the unit root process, the annuity and mortgage contracts have a higher value when the initial short-term interest rate is 4 percent rather than 12 percent. This counter-intuitive result can be explained with the help of Table 7.3 which shows that the initial spreads between the mortgage rate and the short-term interest rate differ substantially. For example, the spread at a short-term interest rate of 4% is twice as large as at an initial spot rate of 8%. Compared with the spread at 12%, it is even 4.5 times larger. Due to the callable features, the mortgages are less sensitive to the spread differences than the noncallable annuity contract.

When the stationary process is utilized, the high initial spread at a short-term interest rate of 4 percent does not lead to relative high contract values because the short-term interest rate is pulled upwards to its unconditional mean.

The impact of prepayment costs

In Table 7.2 the up-front costs of setting up a new mortgage loan to replace the existing contract are assumed to equal one percent of the unpaid balance. In Figures 7.3 and 7.4 the mortgage and option values are plotted for alternative up-front costs. In Figure 7.3 the interest rate dynamics are described by a stationary process. Due to the mean-reverting characteristics of this process the value of the prepayment option is low at low interest rates. At higher interest rates the value of the option increases, which makes it more sensitive to the up-front costs. When the unit root process is used, this sensitivity does not depend on the interest rate level. This is illustrated in Figure 7.4, which clearly shows that the impact up-front costs have on the option value hardly depends on the initial level of the interest rate. This can be explained by the fact that due to the absence of a mean-reverting drift, the option value depends only on the interest rate volatility. As a

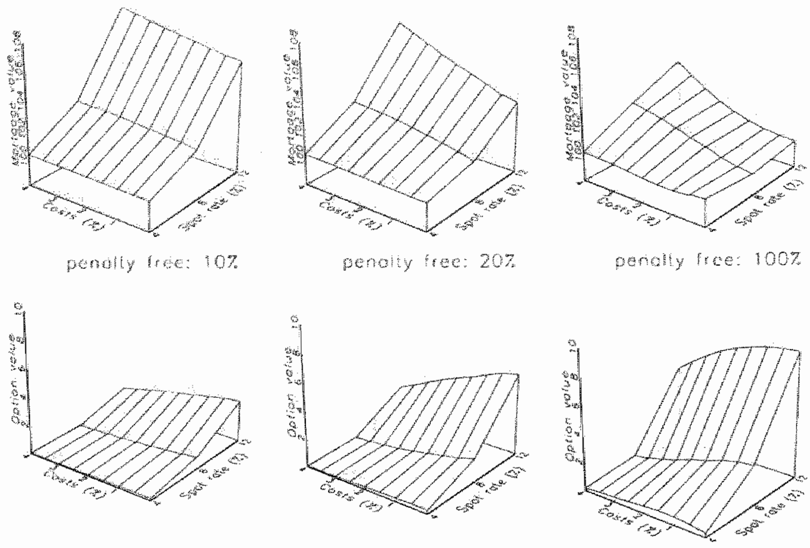

FIGURE 7.3: THE INFLUENCE OF UP-FRONT COST

These figures show the effect different up-front costs have on the mortgage and option value. The impact is illustrated for mortgages with different prepayment restrictions and different initial short-term interest rates. The interest rate dynamics are based on the stationary process.

consequence, the prepayment likelihood at low interest rates is much higher than when the stationary process is used. Since at high interest rates the opposite holds, the value of the prepayment option is less sensitive to the initial interest rate, and also the impact of the up-front costs depends less on this initial rate.

The impact of minimum interest rate guarantees

Preceding a mortgage contract, the borrower will gather information and compare alternative quotation offers. During this time for reflection the mortgage rate might alter, either to the advantage or disadvantage of the borrower. Many Dutch quotation offers contain a guarantee in which the financial institute assures that at the end of the offer period, the borrower may set the contract at the lowest mortgage rate observed during that period. For a quotation offer period of 3 months and for initial spot rates of 4, 8 and 12%, the results are summarized in Table 7.4.

The valuation results of mortgage contracts preceded by a quotation offer can not be directly compared with the mortgages studied in Table 7.2. These latter contracts take effect immediately, while the contracts considered in Table 7.4 are not issued until the end of the offer period. The difference is illustrated in Figure 7.5.

In Figure 7.5, a mortgage contract which immediately takes effect at time t' is issued

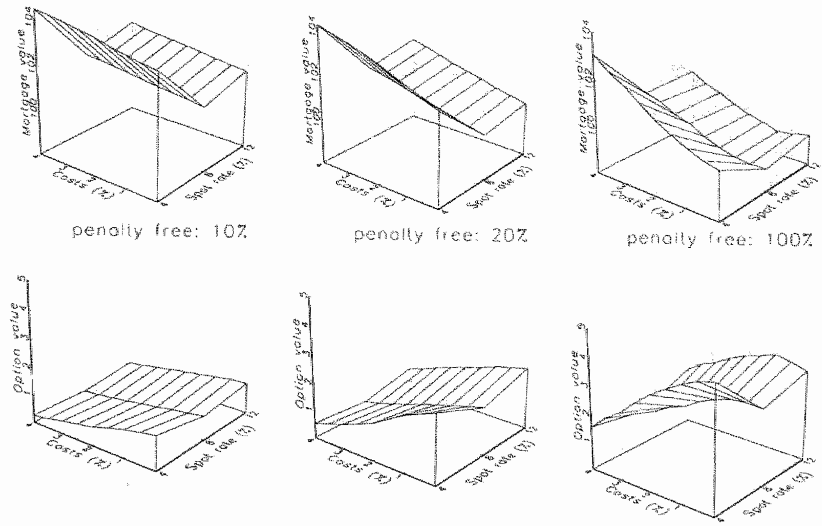

FIGURE 7.4: THE INFLUENCE OF UP-FRONT COST

These figures show the effect different up-front costs have on the mortgage and option value. The impact is illustrated for mortgages with different prepayment restrictions and different initial short-term interest rates. The interest rate dynamics are based on the unit root process.

with a contract rate matching state 2. (Note that state 1 correlates with the highest interest rates.) On the other hand, if a quotation offer precedes the contract, the mortgage will be issued at time t^*. If a minimum interest rate guarantee is embodied in the quotation offer, the contract is set at the lowest mortgage rate observed during the offer period, in which case the contract rate will correspond with state 4. If such a guarantee is not included, the contract rate will be consistent with state 3: the one that occurs at the end of the offer period. Table 7.4 considers these latter two situations, *i.e.*, it reports the differences between the valuation results at time t^* of two contracts which are both preceded by a quotation offer, however, one contains a minimum interest rate guarantee while the other one does not. Only the differences between those two types of contracts are reported, whereby the results of a contract with a minimum interest rate guarantee are subtracted from the results of a contract without this assurance.

Contracts settled at the contract rate of time t^* are expected to yield higher periodical payments than the ones settled at the lowest mortgage rate that occurred during the offer period. Hence, it is not surprising that Table 7.4 shows that the present values of the contracts preceded by an offer without a minimum interest rate guarantee are slightly higher than the values of the contracts which include this guarantee. Since the former contracts are issued at a higher contract rate there is a larger probability that they will

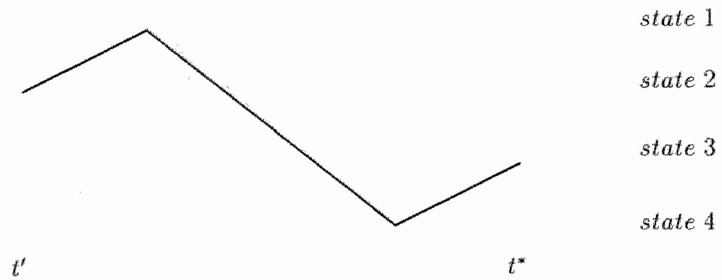

FIGURE 7.5: RANDOM MORTGAGE RATE PATH DURING QUOTATION OFFER PERIOD

TABLE 7.4: VALUES WITH AND WITHOUT A MINIMUM INTEREST RATE GUARANTEE

Spot rate	4%		8%		12%	
VAR spec.	stationary	unit root	stationary	unit root	stationary	unit root
			0% penalty-free prepayment			
Annuity value	0.83	0.74	1.58	1.36	0.65	0.85
Dur. annuity	-0.13	-0.11	-0.25	-0.22	-0.09	-0.13
Δ annuity	0	0	0	0	0	0
			10% penalty-free prepayment			
Mortgage value	0.76	0.60	1.21	1.16	0.40	0.67
Dur. mortgage	-0.98	-0.88	-2.03	-1.38	-0.68	-0.96
Δ mortgage	-0.02	0	0	0	0	0
Option value	0.06	0.15	0.36	0.20	0.27	0.18
			20% penalty-free prepayment			
Mortgage value	0.70	0.45	0.85	0.93	0.10	0.56
Dur. mortgage	-1.82	-1.64	-3.92	-2.71	-1.44	-1.75
Δ mortgage	0	-0.06	-0.02	-0.01	0	-0.01
Option value	0.10	0.28	0.71	0.43	0.55	0.30
			100% penalty-free prepayment			
Mortgage value	0.47	0.16	0.27	0.66	0.05	0.38
Dur. mortgage	-4.38	-3.42	-7.88	-5.92	-1.79	-3.95
Δ mortgage	-0.03	-0.02	-0.05	-0.04	-0.02	-0.03
Option value	0.36	0.58	1.31	0.69	0.71	0.49

The table shows the differences between two contracts which are both preceded by a quotation offer. The results of a contract with a minimum interest rate guarantee are subtracted from the results of a contract without this assurance. Utilized interest rates: Jan. 1972 - Dec. 1995, up-front fee: 1% of the loan, quotation offer guarantee: 3 months. *Option value* reflects the value of the prepayment option.

be prepaid. This is reflected by the negative sign of the difference in duration, *i.e.*, the duration of a contract without a minimum interest rate guarantee is smaller than the duration of a contract with this a guarantee.

Moderate and high starting rates yield similar results. At an initial short-term interest rate of 8% the drift term is less emphatically present in the interest rate dynamics, while starting with a spot rate of 12% results in a negative drift which pulls both the discount and mortgage rate down. This negative drift increases the probability that the minimum mortgage rate is observed at the end of the offer period, such that the importance of a minimum interest rate guarantee decreases.

The results in Table 7.4 show that a minimum interest rate guarantee can always be offered at very low costs. Table 7.5 explains why. Because of the lower contract rate, prepayment is less likely if a minimum interest rate guarantee is included in the offer preceding the contract. Consequently, the prepayment option reduces in value.

TABLE 7.5: SUBSTITUTION EFFECT

VAR specification	stationary		unit root	
Quotation guarantee	no	yes	no	yes
	Spot rate: 4%			
Duration mortgage	26.99	31.36	19.80	23.22
Option value	0.54	0.18	4.02	3.44
	Spot rate: 8%			
Duration mortgage	18.97	26.85	22.09	28.01
Option value	3.80	2.49	3.49	2.80
	Spot rate: 12%			
Duration mortgage	7.31	9.09	17.77	21.72
Option value	9.57	8.85	3.87	3.38

This table shows the substitution effect between the quotation offer guarantee and the prepayment option. The table reports the duration and option values of a fully callable mortgage contract preceeded by a three months' offer. *Quotation guarantee* indicates whether or not a minimum interest rate guarantee is included in this offer. Utilized interest rates: Jan. 1972 - Dec. 1995, up-front fee: 1% of the loan. *Option value* reflects the value of the prepayment option.

In conformity with Table 7.2, Table 7.5 shows that in the unit root setting the duration and option value hardly fluctuate between the different initial short-term interest rates. Due to the absence of a mean-reverting drift term, prepayment is equally likely at low and high starting rates. This partially explains why no pattern can be recognized in the results based on the unit root process shown in Table 7.5. The other main cause is the fact that the historical spread between mortgage rates and short-term interest rates varies with interest rate levels.

Table 7.5 reports the duration of the mortgage contract and the value of the prepayment option of a contract which is preceded by a three months' offer. If the maturity of this offer is lengthened, the interest rates described by the stationary process are further converged to their long-term rates when the period ends. For low initial spot rates this results in a greater difference between the contracts with and without a minimum interest rate guarantee. At an initial spot rate of 8%, this fluctuation is much smaller. Lengthening the quotation offer period when the starting short-term interest rate is 12% reduces the prepayment likelihood and results in higher duration measures and lower option values. This holds true for both contract types. However, a contract which is preceded by an offer that embodies the guarantee shows a stronger reaction. The differences between both types of contracts therefore increase as the offer periods lengthens.

In case the unit root model is utilized, a longer quotation offer simply means a larger probability of a low contract rate. This reduces the prepayment likelihood which causes the option value to decrease and the duration to increase.

7.6 Sample period sensitivity

The mortgage valuation model developed in Chapter 5 is based on various specifications of a one-factor interest rate model. The sample period in Chapter 5 was January 1981 through December 1994, while the VAR parameters which have been used in this chapter so-far are based on observations between January 1972 and December 1995. In order to compare the multi-factor approach with the one-factor models, the VAR's have been re-estimated for the period January 1981 through December 1994.[7]

The impact of the shortened period on the results of the multi-factor model can be seen by comparing Tables 7.1 and 7.2 with Tables 7.6 and 7.7. The stationary process now shows lower interest rates and standard deviations, *i.e.*, the unconditional means of the short-term, long-term and mortgage interest rates are 6.27%, 7.32% and 8.23%, respectively. In other words, approximately 50 to 60 basis points less than before. Figure 7.6 plots the average mortgage rate path resulting from 30,000 simulation runs based on VAR analyses of Dutch interest rates between 1981 and 1994.

The most remarkable change in a contract value occurs at the spot rate of 4%, where the annuity value is no longer above par when based on the stationary process. Despite the lower unconditional mean of the short-term interest rate, the infinite horizon discount

[7] Besides the VAR parameters, the risk adjustment factors are re-adapted for this change in research period. The need to adjust the initial spreads between the long-term and short-term interest rate, and between the mortgage rate and short-term interest rate was also checked. These initial spreads are derived from the observed interest rates and determine the starting rates in the simulations. Hence, they influence the path that the interest rates follow afterwards. The spreads that hold for the whole 1972-1995 period at short-term interest rates of 4 and 12% were also found to hold for the subperiod between 1981 and 1994. The spreads only needed a small adjustment at a short-term interest rate level of 8%. The spread between the long-term and short-term interest rate decreased by 14.5 basis points from 119.5 to 105, while the spread between the mortgage rate and the short-term interest rate increased from 140.5 basis points to 165 basis points.

TABLE 7.6: DESCRIPTIVE STATISTICS OF THE INTEREST RATES

VAR specification	stationary			unit root		
Initial spot rate	4%	8%	12%	4%	8%	12%
			After 60 months			
Short-term interest rate	6.13	6.40	6.67	5.23	7.85	10.81
Standard deviation	1.77	1.76	1.77	2.65	3.16	3.26
Long-term interest rate	7.24	7.48	7.69	6.13	8.72	11.59
Standard deviation	1.22	1.22	1.22	2.57	2.89	2.92
Mortgage rate	8.14	8.38	8.61	7.07	9.66	12.55
Standard deviation	1.19	1.19	1.19	2.53	2.87	2.91

The table shows the average rates and standard deviations of the short-term, long-term and mortgage interest rates at the end of the 60 months' fixed-rate period. The results are derived by 30,000 simulation runs based on VAR analyses of Dutch interest rates between 1981 and 1994.

rate converges to the same 8% as imposed by the market price of risk adjustment. The risk premium derived from interest rates observed between 1981 and 1994 is larger than the one based on the entire 1972-1995 period. The average discount rate paths shown in Figure 7.7 include these risk premiums. Even though both paths eventually converge at an 8% level, the average discount factor during the first 60 months differs substantially. Due to the on average higher discount factor, the annuity contract has a lower value in the 1981-1994 setting than in the 1972-1995 perspective.

For the stationary process, Table 7.2 reported a relative high duration at an initial spot rate of 8%. Especially for a fully callable mortgage this measure stood out. In Table 7.7 this is no longer the case. Here the duration decreases with increasing interest rates, also when mortgages can completely be called without penalty. The explanation for this difference can be found by comparing the lower-middle diagrams of Figures 7.2 and 7.6. When the parameters are based on the entire 1972-1995 period, the average mortgage rate path increases for the first 9 months, while only after 39 months the mortgage rate is back at its starting level. For the subperiod 1981-1994, these figures are 3 and 5 months, respectively. Hence, the prepayment likelihood is much larger in this latter case, as reflected by the lower duration measure and higher option value. Not only the duration, but also the Δ risk measure alters when the length and placement of the sample period for the estimation of interest rate dynamics changes.

Similar to Table 7.2, Table 7.7 shows that the valuation results based on the unit root process are not in conformity with economic expectations. For example, the value of the annuity and mortgage contracts decrease with increasing initial interest rates. The same holds true for the value of the prepayment option and also the duration measure does not reveal a consistent pattern.

TABLE 7.7: MORTGAGE VALUES: SAMPLE PERIOD JAN. 1981 - DEC. 1994

VAR specification	stationary			unit root		
Spot Rate	4%	8%	12%	4%	8%	12%
	0% penalty-free prepayment					
Annuity value	99.51	103.49	108.47	105.14	104.22	102.77
Duration annuity	50.28	48.42	46.51	50.67	48.38	45.81
Δ annuity	1.55	1.48	1.42	0.83	0.79	0.75
	10% penalty-free prepayment					
Mortgage value	99.45	102.50	105.48	104.03	103.09	101.77
Duration mortgage	46.75	40.56	35.27	45.04	42.86	40.28
Δ mortgage	1.47	1.34	1.22	0.79	0.75	0.71
Option value	0.06	0.99	2.99	1.11	1.14	1.00
	20% penalty-free prepayment					
Mortgage value	99.39	101.53	102.49	102.91	101.97	100.74
Duration mortgage	43.17	32.42	23.36	39.14	37.10	34.54
Δ mortgage	1.39	1.20	1.01	0.74	0.71	0.67
Option value	0.12	1.96	5.98	2.23	2.25	2.03
	100% penalty-free prepayment					
Mortgage value	99.30	99.87	99.69	100.68	100.02	99.25
Duration mortgage	27.05	14.13	5.40	21.94	22.20	20.24
Δ mortgage	0.96	0.70	0.36	0.52	0.53	0.50
Option value	0.20	3.62	8.78	4.45	4.19	3.53

The table shows valuation results based on alternative VAR specifications, initial short-term interest rates, and penalty-free prepayment conditions. Duration is expressed in months. The *effective* duration Δ is defined as the semi-elasticity of the mortgage with respect to the short-term interest rate. A quotation offer guarantee is not included in the mortgage contracts. Utilized interest rates: Jan. 1981 - Dec. 1994, up-front fee: 1% of the loan.

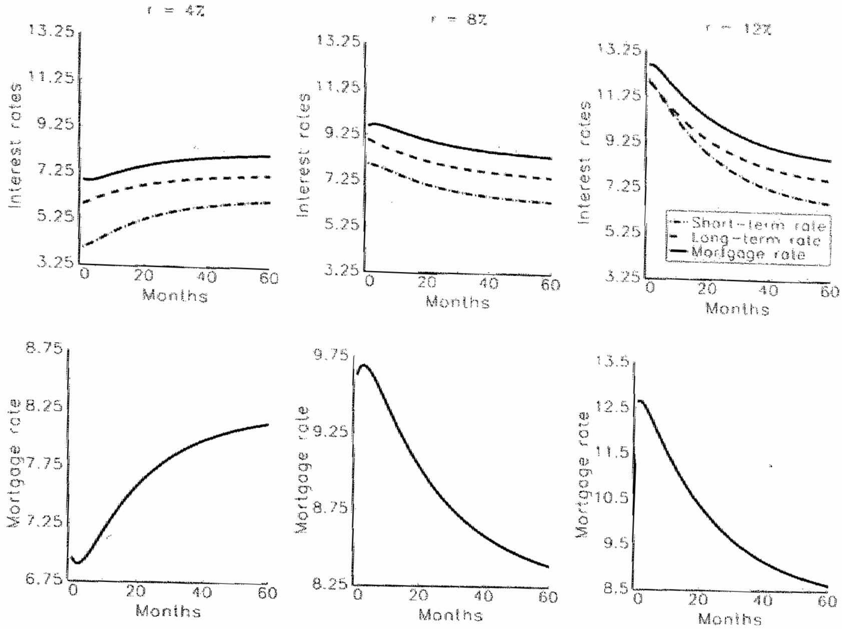

FIGURE 7.6: AVERAGE MORTGAGE RATE PATH

The upper diagrams of this figure show the average interest rate path generated by the stationary VAR model. The lower diagrams yield a close up look at the mortgage rates. The r value on top of each column represents the short-term interest rate at which the simulations started. The results are derived by 30,000 simulation runs based on VAR analyses of Dutch interest rates between 1981 and 1994.

7.7 One-factor models versus multi-factor models

This section compares the results of the valuation procedure based on the one-factor models developed in Chapter 5 with those of the multi-factor approach. Since Chapter 5 concentrates on noncallable annuities and fully callable mortgage contracts we focus solely on the same here.

The left-hand side of Table 5.6 in Chapter 5 shows the valuation results based on three one-factor interest rate models where mortgage rates are exogenously related to short-term interest rates. This exogenous specification is empirically determined based on historical observations in the Netherlands. The results reported in Table 5.6 are based on the same prepayment rule as applied in this chapter.

Table 7.8 presents the changes in the results when a single factor model underlying the pricing algorithm is replaced with a multi-factor interest rate process. The figures are found by subtracting the results of the multi-factor model from those of the various one-factor models. Note that the spreads between the short-term interest rate and the mortgage rate

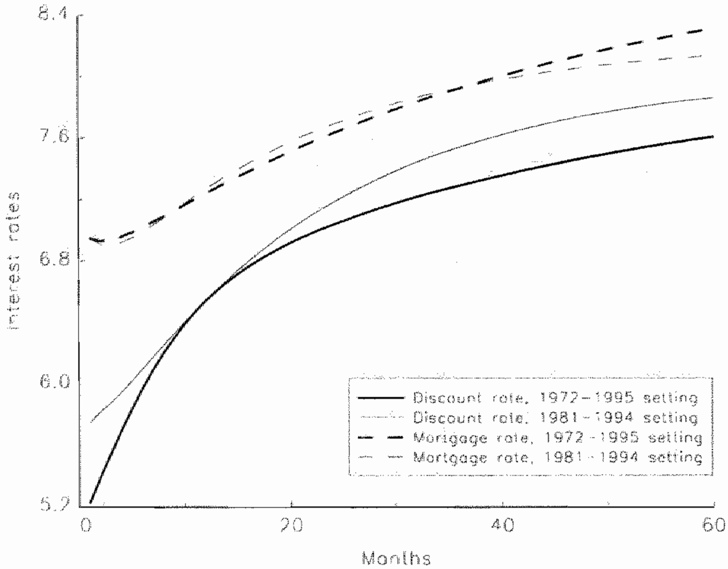

FIGURE 7.7: AVERAGE MORTGAGE RATE PATH

The bold lines in this figure show the average discount and mortgage rate paths resulting from 30,000 simulations based on the stationary specification of the VAR model which took interest rates observed between 1972 and 1995 as input. The thin lines are based on interest rates observed between 1981 and 1994. The discount rate is equal to the short-term interest rate adjusted for the market price of risk. The initial short-term interest rate equals 4%.

are not the same in both settings. At an initial spot rate of 4%, the one-factor models utilize a spread which is 70 basis points higher than its counterpart in the multi-factor settings. At a spot rate of 8%, the spread in the one-factor models is 53 basis points less than the spread applied in the multi-factor model, while at an initial spot rate of 12% it is 48 basis points higher.[8]

In Chapter 5 as well as in this chapter, we use the effective duration measure to analyze the impact of a small adjustment in the short-term interest rate on the mortgage value. In Chapter 5, the distance between successive interest rate grid points determined the size of this small adjustment. Consequently, adjustments of 10 basis points were studied. In the simulation approach we are free to choose a particular adjustment. The Δ measures reported in this chapter are based on deviations in the short-term interest rate of one basis point, *i.e.*, the variable h in Equation (7.10) is equal to one basis point. To bring the Δ measures of both chapters in line with one another we also calculated the effective duration

[8]In Chapter 5 we fitted the function $y(t) = f((r(t))$, where $y(t)$ is the mortgage rate and $r(t)$ is the short-term interest rate. In this chapter we calculate the average spreads at spot rates of 4, 8 and 12% rather than estimating the entire functional relation.

Δ based on deviations of 10 basis points. This hardly changed the Δ measure, so that the interest rate risk benchmark reported in Table 7.7 can be compared with its counterpart in Table 5.6.

TABLE 7.8: ONE-FACTOR MODELS VERSUS MULTI-FACTOR MODELS

VAR specification	stationary			unit root		
Spot rate	4%	8%	12%	4%	8%	12%
	CIR model - multi-factor model					
Annuity value	6.24	0.58	1.65	0.61	-0.15	7.35
Duration annuity	-0.56	-0.20	-0.89	-0.95	-0.16	-0.19
Δ annuity	0.62	0.59	0.52	1.34	1.28	1.19
Mortgage value	5.08	0.42	0.57	3.70	0.27	1.00
Duration mortgage	14.11	2.25	0.45	19.22	-5.82	-14.39
Δ mortgage	-1.97	-0.56	-0.45	-1.53	-0.39	-0.59
Option value	1.18	0.16	1.08	-3.07	-0.41	6.33
	Nonlinear model - multi-factor model					
Annuity value	7.27	-0.41	0.93	1.64	-1.14	6.63
Duration annuity	-0.44	-0.27	-0.99	-0.83	-0.23	-0.29
Δ annuity	-1.29	2.49	-0.69	-0.57	3.18	-0.02
Mortgage value	7.26	-0.67	0.54	5.88	-0.82	0.98
Duration mortgage	21.30	4.88	-1.65	26.41	-3.19	-20.24
Δ mortgage	-0.94	0.72	-0.39	-0.50	0.89	-0.53
Option value	0.02	0.26	0.40	-4.73	-0.31	5.65
	Nonparametric model - multi-factor model					
Annuity value	0.74	0.68	6.30	-4.89	-0.05	12.00
Duration annuity	-0.88	-0.18	-0.51	-1.27	-0.14	0.19
Δ annuity	-1.31	-0.06	-0.57	-0.59	0.63	0.10
Mortgage value	0.76	-0.40	0.39	-0.62	-0.55	0.83
Duration mortgage	9.85	-2.90	-1.32	14.96	-10.97	-16.16
Δ mortgage	-0.75	-0.13	-0.28	-0.31	0.04	-0.42
Option value	-0.01	1.07	5.91	-4.26	0.50	11.16

The numbers in this table are found by subtracting the results of the multi-factor model from those of the various one-factor models. The single factor models are based on the empirical relation between short-term interest rates and mortgage rates in the Netherlands.

Table 7.8 shows that the effects of the alternative interest rate processes on the mortgage values are minor. Except at an initial spot rate of 4%, the difference between the mortgage valuation results based on a single or multi-factor interest rate model does not exceed 1%. However, one may not conclude that the underlying interest rate process is therefore unimportant. The Δ measure, which we saw earlier was sensitive to the period under consideration, turns out to be very responsive to the interest rate process as well. The interest rate sensitivity is largest when a multi-factor approach is used.

As previously mentioned, the stationary multi-factor process prescribes that the discount rate will converge at the same 8% level as in the single factor specifications. Differences in the valuation results are therefore mainly caused by spread differences between the short-term interest rate and the mortgage rate, and by differing mean-reversion drifts and deviating interest rate volatilities.

The unit root process requires an alternative approach to include the market price of interest rate risk. As a consequence, the dynamics of the discount rate in the single factor approaches differ substantially from the dynamics described by the multi-factor unit root specification. This difference complicates the comparison of the results.

Chapter 5 also considers an optimal prepayment rule which states that the borrower prepays when the value of the mortgage, if left uncalled, exceeds the outstanding debt plus any transaction costs associated with refinancing. As discussed in Chapter 5, this prepayment rule can only be used when the mortgage rate is endogenously determined as a function of the short-term interest rate. This endogenous relation is derived by imposing that the mortgage value at origination equals the face value of the loan. Even though this requirement is not imposed on the multi-factor pricing algorithm, we can compare both approaches because the multi-factor approach yields mostly mortgage values close to par. Table 7.9 presents the differences between the results of both approaches. Once again, Table 7.9 focuses on noncallable annuities and fully callable mortgage contracts and the figures represent the differences between the valuation results of both approaches.

Table 7.9 displays that the mortgage and annuity values found by the alternative pricing algorithms are very similar, but that the risk measures differ strongly. When noncallable annuities are studied the fluctuations in the duration are rather small, but when fully callable contracts are analyzed, this measure alters substantially, especially at low rates. In absolute numbers, the Δ measure is much smaller than the duration. Consequently, the deviations in the Δ measure stand out less, even though the proportional mutation is much larger. Both Tables 7.8 and 7.9 exhibit the impact which the underlying interest rate model has on risk measures and therefore on the required hedging strategy.

7.8 Conclusion

Valuation in Chapter 5 proceeds through a discrete/finite state Markov chain, for which three empirical one-factor dynamic processes for the short-term interest rate are specified. The fundamental variable in these single factor models is the instantaneous interest rate which is assumed to be the sole factor influencing mortgage rate dynamics. The results of Chapter 6, however, indicate that the mortgage rate dynamics can be modelled more accurately when more factors are considered. We therefore include the long-term interest rate and the mortgage rate as two additional factors in this chapter. The dynamic interactions between these variables are described by the Vector AutoRegressive processes

TABLE 7.9: ONE-FACTOR MODELS VERSUS MULTI-FACTOR MODELS

VAR specification	stationary			unit root		
Spot rate	4%	8%	12%	4%	8%	12%
	CIR model - multi-factor model					
Annuity value	4.05	1.47	0.06	-1.58	0.74	5.76
Duration annuity	-0.24	-0.33	-0.68	-0.63	-0.29	0.02
Δ annuity	0.63	0.58	0.52	1.35	1.27	1.19
Mortgage value	0.70	0.13	0.31	-0.68	-0.02	0.75
Duration mortgage	3.10	-5.31	-1.86	8.21	-13.38	-16.70
Δ mortgage	-0.91	-0.52	-0.16	-0.47	-0.35	-0.30
Option value	3.36	1.34	-0.25	-0.89	0.77	5.00
	Nonlinear model - multi-factor model					
Annuity value	3.29	2.22	-1.48	-2.34	1.49	4.22
Duration annuity	0.13	-0.66	-0.65	-0.26	-0.62	0.05
Δ annuity	-1.29	2.46	-0.69	-0.57	3.15	-0.02
Mortgage value	0.70	0.13	0.31	-0.68	-0.02	0.75
Duration mortgage	-3.82	-0.21	-2.92	1.29	-8.28	-17.76
Δ mortgage	-0.87	-0.20	-0.24	-4.36	-0.03	-0.38
Option value	2.60	2.09	-1.79	-1.65	1.52	3.46
	Nonparametric model - multi-factor model					
Annuity value	2.61	3.33	5.44	-3.02	2.60	11.14
Duration annuity	-1.17	-0.57	-0.40	-1.56	-0.53	0.30
Δ annuity	-1.31	-0.06	-0.57	-0.59	0.63	0.10
Mortgage value	0.70	0.13	0.31	-0.68	-0.02	0.75
Duration mortgage	-7.14	-5.23	-1.70	-2.03	-13.30	-16.54
Δ mortgage	-0.83	-0.46	-0.25	-0.39	-0.29	-0.39
Option value	1.92	3.20	5.13	2.33	2.63	10.38

The figures in this table are found by subtracting the results of the multi-factor model from those of the various one-factor models. The single factor models are based on the endogenously derived relation between short-term interest rates and mortgage rates in the Netherlands, whereby it is assumed that the mortgagor prepays the contract when the value of the contract, if left uncalled, exceeds the outstanding debt plus any transaction costs.

developed in the previous chapter. The resulting VAR-parameters are applied to simulate the short-term, long-term and mortgage interest rates which are used as the input of the valuation procedure.

Two specifications of the VAR model are utilized to describe the dynamic interactions between the interest rates. In the first specification a stationary process is analyzed, while the second one includes a unit root. The value of the prepayment option turns out to be especially sensitive to the chosen VAR specification underlying the mortgage pricing algorithm. The up-front costs of starting a new contract also have a substantial impact on the value of the prepayment option. Minimum interest guarantees frequently embodied in quotation offers preceding a Dutch contract seems to only have a small effect on the value and interest rate characteristics of a mortgage contract.

There is tendency to believe that the prepayment option embodied in Dutch contracts is of minor importance in comparison to the United States because of the annual prepayment restrictions. Although less important than in the US, the valuation results clearly show that the limited prepayment option is not insignificant, because the value of a 10% penalty-free prepayment option equals one quarter of the value of a prepayment option without any limitations. The value of the 20% penalty-free prepayment option is even equal to half the value of the 100% penalty-free prepayment option!

The valuation results from the unit root specification of the VAR model are often not in keeping with general economic theory. For example, instead of an increasing value of the annuity and mortgage contract with higher initial interest rates, exactly the opposite is observed. We would also expect to see a pattern in the duration and option value correlating with interest rate fluctuation. However, no such pattern can be detected.

In order to bring the multi-factor approach in line with the one-factor models of Chapter 5, the VAR analyses, which form the basis of the pricing algorithm, are repeated with interest rates observed between January 1981 and December 1994. This adjustment in the considered period turns out to have a major impact on the value of the risk measures of the contract, *i.e.* the effective duration Δ increases substantially when the sample period is shortened.

Mortgage valuation algorithms based on both multi-factor interest rate processes yield mortgage values which are very similar to the results based on the one-factor models. However, the interest rate sensitivity of a mortgage contract is substantially influenced by the choice of a particular interest rate model. As a result, the required hedging strategy will depend on which model has been chosen.

Chapter 8

A repeat sales index for residential property in the Netherlands

8.1 Introduction

For most individuals, the purchase of a house is the largest investment of a lifetime. To analyze the risk and return characteristics of this investment we develop an index for residential property in the Netherlands. This index will show the general price movements in the Dutch property market between May 1973 and December 1995. The index is of interest to institutional investors, tax authorities, academic researchers, and it will help homeowners with their personal finance decisions. Risk and return properties of the housing market determine the diversification possibilities and therefore the composition of homeowners' portfolios. Moreover, the index enables mortgagees to undergo quantitative research regarding default risk.[1]

A residential price index should give a representative image of the tendency of the housing market and has to reflect the fluctuations in house prices rather than the changing composition of houses sold in the different periods. The characteristics of the houses sold change from period to period. Consequently, indices based on summary statistics like the mean and median price of all properties sold in a given period have serious drawbacks.

Methodologies used to construct a constant quality house price index can be divided into three major categories: the hedonic method, the repeat sales method and the hybrid method. In this paper we focus on the repeat sales method introduced by Bailey, Muth and Nourse (1963) and refined by Case and Shiller (1987).

The remainder of this paper is organized as follows. In Section 8.2 we briefly describe the alternative methodologies for constructing a constant quality real estate index. The weighted repeat sales method as presented by Case and Shiller (1987) will be explained in more detail in Section 8.3. The house price data used in this paper will be summarized in

[1] Along with the index constructed here, it is necessary to obtain historical data to research default behavior in the Netherlands. As discussed in Chapter 2 such data are unfortunately hard to come by in the Netherlands.

Section 8.4. The results follow in Section 8.5. The conclusions and recommendations for future research are summarized in Section 8.6.

8.2 Index methodologies

Factors that influence the price of residential property can be divided into two major categories. The first category encompasses property characteristics, the most important of which is probably the location of the house. Property size, age and condition are also included within this category. The second category is made up of economic factors, such as the mortgage rate and inflation. The first category is the starting point for a real estate index constructed with a hedonic regression method, while an index based on a repeat sales regression focuses directly on market factors. Both methodologies are combined in the hybrid method.

Originally, the hedonic technique was developed by Court (1939) for the automobile market. After the theoretical elaboration by Rosen (1974), it was widely applied to the real estate market. The basic idea behind the hedonic method is that the price of a house consists of a combination of property attribute prices, like lot size, number of rooms, the existence of a garage, et cetera. In the hedonic regression technique, the observed sale prices are regressed on these property characteristics. Hereby it is assumed that the prices of these attributes remain constant over time. This is especially a shortcoming when the real estate index covers an extensive period of time. The coefficients resulting from the hedonic regression indicate the extent to which each of the attributes contribute to the total price of the house. A constant quality residential property index can be derived from the hedonic regression results if the coefficients are allowed to be time-heterogeneous.

A hedonic regression will only yield an unbiased index if the correct set of explanatory variables and the correct mathematical relation between these regressors are used. In the exceptional case that these requirements are met, it will still be difficult to execute the regression because the required data are often not available. Additionally, the hedonic method does not use all information efficiently. A single property which is sold twice contains information which can be used more efficiently than is done by a hedonic regression. Most attributes will not have changed in between the two transaction dates of such a repeat sales pair. The difference in price will only be affected by the attributes that did change and the elapsed time between transactions. The repeat sales method utilizes the available information of these repeated transactions much more efficiently.

The data required to run a repeat sales regression consists of prices from properties that are sold more than once in the observed period. Using this sample allows the calculation of price changes for each individual house. The difference between the natural logarithm of the selling and the buying prices is regressed on a set of dummy variables, one for each time period in the sample except for the base period. These dummy variables can take three values: the dummy corresponding to the first sale of the house is -1, and the second sale dummy equals +1. In all other cases the dummy variable is zero. The repeat sales index is found by computing the exponent of the estimated coefficients.

The repeat sales method controls for house quality by using the information from property specific price fluctuations. After all, it is the same house that is looked at. It is hereby assumed that the quality is constant during the intra-sale interval such that renovations between sales are ignored. Homes sold after long time intervals are less likely to meet this assumption. Despite this, these homes have relatively more weight in the original repeat sales method than homes sold over short time intervals. Case and Shiller (1987) attempt to control for this by applying a Generalized Least Squares technique instead of the Ordinary Least Squares method proposed by Bailey, Muth and Nourse (1963).

If the residential property prices follow a random walk, the three stage least squares technique suggested by Case and Shiller (1987) will correct for any heteroskedasticity in the data. After running the Ordinary Least Squares (OLS)regression as proposed by Bailey, Muth and Nourse (1963), the second stage consists of an OLS regression of the squared residuals on a constant and the time between sales. Subsequently, a weighted least squares regression is run such that the weights are the estimated standard deviations of the second stage. The exponent of the estimated coefficients from the third stage yields the weighted repeat sales index.

Alongside heteroskedasticity in the data, a second source of inefficiency in the repeat sales method shows up in the large number of sales that are required before a reasonable repeat sales sample is obtained. Instead of using all available transaction data, only a small portion is used. If this subsample is not representative for the entire property population, the index will be biased. Relatively new houses, and those with undesirable features, are not likely to be included in the repeat sales sample, while so-called starter homes might be over-represented. Clapp, Giaccotto and Tirtiroglu (1991) argue that this does not influence the index. They state that all properties in a given area should appreciate at approximately the same rate. If starter homes appreciate at a faster rate than other properties, these latter properties will become more in demand. As a result, the prices of these properties will be driven up and the discrepancy will disappear.

The repeat sales method assumes that no house attribute changes between transactions. Strictly speaking, no transaction pair can ever meet this assumption because the age of the property will be higher at the second sale. The negative effect of aging implies that a repeat sales index based on homes whose only changing attribute is the increased age will be biased downwards, and thus systematically lower than a pure price index. On the other hand, correcting the index for all depreciation is not advisable either. Depreciation of a property is caused by both physical deterioration and changing market preferences. Homeowners can control most of the physical deterioration and therefore, the property index needs no adjustment. If we assume that houses are well maintained, only general factors that influence the whole market will cause depreciation. A real estate index constructed to show the general tendency of the housing market should thus include those market factors such that a correction for this aspect is undesirable.

In addition to renovation, subsample and depreciation bias, a fourth source of potential bias in the repeat sales index may arise when a geometric mean is used instead of an arithmetic mean. The coefficients estimated by the repeat sales regression are arithmetic means of logged growth rates. The repeat sales index is found by taking the exponent of these coefficients. This logarithmic transformation results in an index which is based on geometric means, making it more difficult to interpret. The value of a real estate portfolio is equal to the sum of the values of the individual properties rather than the product of the separate houses. In other words, portfolio values are arithmetic transformations of prices and not geometric transformations. A real estate index based on arithmetic means is therefore more preferable than an index based on geometric means. Unfortunately, the index based on the repeat sales methodology is a geometric index. Since a geometric mean will always be less than or equal to an arithmetic mean, the repeat sales index is downward biased.

Finally, financial indices are often used as a benchmark for performance measurement. The repeat sales index is not suited for this. Nor can the index be used to settle derivative contracts, the reason for which can be found in the so-called "*revision volatility*". As time advances, new information becomes available: houses that were not embodied in the original subsample might now be sold for a second time and therefore be included in the repeat sales data set. This extra information about the first and second sale of those houses will influence both the current and the past index numbers. Consequently, settling contracts and awarding bonuses to investors on the basis of repeat sales index numbers may result in retraction when new information comes available.

In 1991, Case and Quigley presented a hybrid method which uses all available information and utilizes the controls inherent in repeat transactions. To achieve this, they divide the total data set into three subsets. The first subset contains information on property sold only once, the second embodies data regarding repeat transactions of properties where attributes did not change between both sales dates, and the third includes repeat transactions of properties whose attributes did change. For each of these subsets a separate regression equation is set up: the first being a pure hedonic equation, the second being a pure repeat sales equation and the third being a modified repeat sales equation. In fact this is a system of three seemingly unrelated regression equations as described by Zellner (1962).

By combining the hedonic and repeat sales technique, the hybrid method includes both the desirable and undesirable features of the two approaches. It increases the efficiency of the repeat sales estimator at the cost of potential specification bias in the hedonic part of the system. Two other shortcomings of the hedonic model come into play here as well. The first being the assumption that the prices of the property attributes remain constant over time, and the second being the lack of data required to estimate the parameters.

In contrast to Case and Quigley (1991), Case, Pollakowski and Wachter (1991) found that using the hybrid method instead of the hedonic or repeat sales methods does not increase efficiency. By comparing the hedonic and repeat sales index they found that the latter index displays smaller increases in house prices than the former. A similar result was found by Mark and Goldberg (1984) and by Cannaday and Yang (1995). The empirical results of Cannaday and Yang (1995) support the hypothesis that a repeat sales index is downward biased if no correction for depreciation takes place. In disagreement with this, Gatzlaff and Ling (1994) found that indices based on median, repeat sales and hedonic methods all produce good estimates of the property index. Clapp, Giaccotto and Tirtiroglu (1991) perceived that over long periods of time (they suggest three years or more) a repeat sales index does not systematically differ from an index based on all available transaction data. Abraham and Schauman observe in addition to these mixed results, that the weighted repeat sales index based on the data set from the Federal Home Loan Mortgage Corporation (Freddie Mac) behaves like other indices up until 1985. After that the index shows a stronger growth than the others. The probable consensus of all these studies is found by Gatzlaff and Haurin (1996), who conclude that a pure repeat sales index is biased upward during a period of economic growth and downward during a period of economic weakness.

This literature review shows that empirical studies disagree about which methodology is preferable above all others. Although theoretically superior, the hedonic and hybrid methods did not necessarily turn out to be better than the repeat sales technique. Furthermore, both the hedonic and hybrid method are hard to implement. In order to obtain unbiased estimates of a property index it is essential that the hedonic regression equation is correctly specified with respect to both functional form and regressors. This factor, along with the limited availability of data, prompted us to choose the weighted repeat sales methodology in constructing a residential property index for the Netherlands.

8.3 Weighted repeat sales methodology

In this section we will describe the computations involved with the weighted repeat sales method. Basically, a Generalized Least Squares (GLS) estimation is employed with weights that are related to the time between two consecutive transactions of one property unit.

Suppose that we have data on repeat sales for a sample period of length T. Let P_{τ^k} be the property value at time $\tau \in \{1,\ldots,T\}$. The superscript k on the time index refers to the k^{th} repeat sale ($k=1,\ldots,K$). A complete repeat sale comprises a pair $\{P_{\tau_1^k}, P_{\tau_2^k}\}$. The first number corresponds with the purchase value and the second number coincides with the value of the property at the selling date. In the first step we perform the following regression:

$$\ln P_{\tau_2^k} - \ln P_{\tau_1^k} = \beta_t \left(\delta_{t\tau_2^k} - \delta_{t\tau_1^k}\right) + \varepsilon_k, \qquad \varepsilon_k \sim IID(0, \sigma_\varepsilon^2), \tag{8.1}$$

$$\tau_1^k, \tau_2^k \in \{1,\ldots,T\}, \qquad k=1,\ldots,K, \qquad t=1,\ldots,T,$$

with

$$\delta_{t\tau_i^k} = \begin{cases} 1, & t = \tau_i^k \quad i = 1,2, \quad t = 1,\ldots,T, \\ 0, & \tau_i^k = 1, \quad i = 1,2, \\ 0, & \text{otherwise.} \end{cases} \tag{8.2}$$

In words, the changes in logarithmic prices are regressed on a set of dummy variables that take values -1 and 1 at the times of buying and selling, respectively. The coefficients β_t are the estimates for the logarithmic repeat sales index. The dummy variables for the first period $t=1$ are set to zero for identification.

In earlier studies on repeat sales indices, notably Bailey *et al.* (1963), the error terms ε_k are assumed to have a constant variance. This assumption has been disputed by Case and Shiller (1987). These authors argue that the length of the time interval between sales is related to the variance. In fact, they assume that the logarithm of a particular price consists of three parts: a general level c_τ, an uncorrelated random walk x_{τ^k} and a property-specific random error η_{τ^k}:

$$\ln P_{\tau^k} = c_\tau + x_{\tau^k} + \eta_{\tau^k}, \qquad \eta_{\tau^k} \sim IID(0,\sigma_\eta^2), \tag{8.3}$$

with

$$x_{\tau^k} = x_{\tau^k - 1} + \nu_{\tau^k}, \qquad \nu_{\tau^k} \sim N(0,\sigma_x^2). \tag{8.4}$$

Consequently, Case and Shiller (1987) suggest to run an additional regression, in which estimates for the unknown variances of the idiosyncratic error term and ν_{τ^k} must be found. This regression reads:[2]

$$\hat{\varepsilon}_k^2 = 2\sigma_\eta^2 + \sigma_x^2\left(\tau_2^k - \tau_1^k\right) + \xi_t, \tag{8.5}$$

with σ_η^2 and σ_x^2 the coefficients to be estimated. The GLS estimation procedure is finished by repeating the first-stage regression with the left-hand side variables $\left(\ln P_{\tau_2^k} - \ln P_{\tau_1^k}\right)$ replaced by

$$\frac{\ln P_{\tau_2^k} - \ln P_{\tau_1^k}}{\sqrt{2\hat{\sigma}_\eta^2 + \hat{\sigma}_x^2\left(\tau_2^k - \tau_1^k\right)}}.$$

The dummy variables are adjusted accordingly. The new coefficients β_t^{GLS} are the estimates of the logarithmic weighted repeat sales index.

[2] From equation (8.3) we know that

$$\ln P_{\tau_2^k} - \ln P_{\tau_1^k} = c_{\tau_2} - c_{\tau_1} + \sum_{j=\tau_1}^{\tau_2} \Delta x_{j^k} + \eta_{\tau_2^k} - \eta_{\tau_1^k},$$

with

$$\begin{aligned} \Delta x_{\tau^k} &= \varepsilon_{\tau^k} \qquad \varepsilon_{\tau^k} \sim N(0,\sigma_x^2), \\ \eta_{\tau^k} &\sim IID(0,\sigma_\eta^2), \end{aligned}$$

such that

$$\sigma^2_{(\ln P_{\tau_2^k} - \ln P_{\tau_1^k})} = \left(\tau_2^k - \tau_1^k\right)\sigma_x^2 + 2\sigma_\eta^2.$$

From which equation (8.5) follows directly.

The heteroskedasticity correction of Case and Shiller encompasses the corrections proposed by Bailey *et al.* (1963) for correlated errors. When a particular house has been sold more than twice within the sample period the errors on the price differential are correlated, because of common appearances of sale price errors η_{τ^k}.[3]

The heteroskedasticity correction of Case and Shiller (1987) has been subject to considerable debate.[4] Many of the corrections that have been suggested try to incorporate property-specific characteristics, yielding so-called hybrid property indices. Other approaches focus on the GLS procedure itself. For the US market the Federal Home Loan Mortgage Corporation (Freddie Mac) and the Federal National Mortgage Association (Fannie Mae) have made a weighted repeat sales index in which the second-stage regression is replaced by

$$\hat{\varepsilon}_k^2 = c_0 + c_1\left(\tau_2^k - \tau_1^k\right) + c_2\left(\tau_2^k - \tau_1^k\right)^2 + \xi_t. \tag{8.6}$$

In fact, many functional forms are possible here. It is very difficult to determine *a priori* an economically sensible form. Another way to look at the heteroskedasticity issue, is to model it in a conditional way, as was first proposed by Engle (1982). These AutoRegressive Conditional Heteroskedasticity (ARCH) models have been successfully applied in a wide variety of financial problems.[5] We will leave this subject for further research.

8.4 Data description

The data used to construct the weighted repeat sales index for residential property in the Netherlands was collected by members of the Dutch Association of Real Estate Agents (NVM) from May 1973 to February 1996. Currently, the association arranges around 57 percent of all transactions on the Dutch residential property market. In 1995 the association reported 87,642 sales of the total 154,750 residential transactions recorded by the land registry.

Before the data was used to estimate the repeat sales property index, a correction was made for the differences in the sale conditions. Subsequently, houses which had been sold more than six times during the observed period, and those whose recorded price was less than 10,000 guilders were eliminated.[6] Transaction pairs which yielded average monthly returns above 5 percent were also taken out of the data because in those cases a change in

[3] In order to obtain the Bailey *et. al.* (1963) correction, Equation (8.5) has to be truncated to

$$\hat{\varepsilon}_k^2 = 2\sigma_\eta^2 + \xi_t.$$

[4] See, for example, Wang and Zorn (1995) and the references therein. Goetzmann and Spiegel (1995) add an intercept term to the weighted repeat sales procedure to capture home improvements and other fixed costs. We estimated the repeat sales index both with and without this constant term. Both approaches yield very similar results. In the remainder of this chapter we therefore only present the index based on the original approach proposed by Case and Shiller.

[5] See Bollerslev *et. al.* (1992) for a survey on ARCH models and its applications in finance.

[6] Using the exchange rate of August 5^{th}, 1996 this amount is equal to 6,040 US-dollars.

housing quality most certainly took place. Another reason to exclude these observations is that houses could have been split into smaller dwelling units.

After these adjustments, 228,144 repeat sales pairs were extracted from a total of 1,089,176 transactions which were registered between May 1973 and February 1996, inclusively. The observations of January and February 1996 are included in the repeat sales subsample since they contain additional information used to estimate a repeat sales index covering the period to December 1995. It goes without saying that those observations are excluded in the mean and median indices which run until the end of 1995.

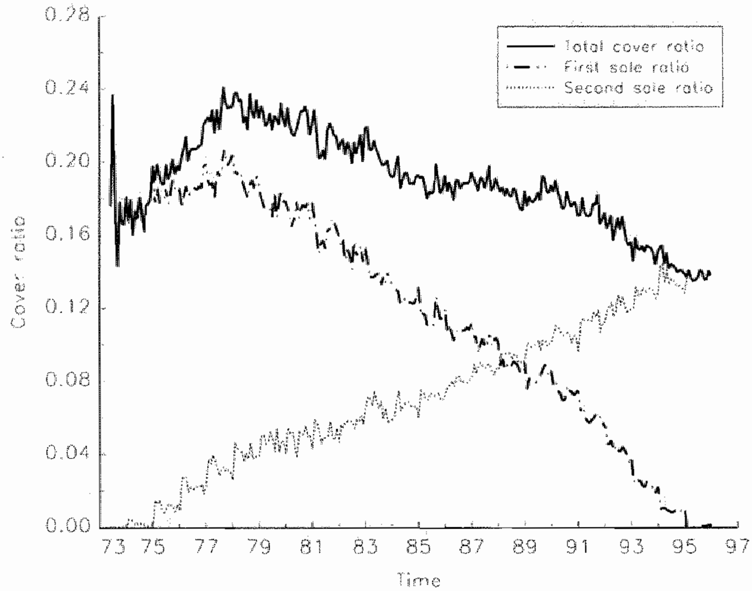

FIGURE 8.1: COVER RATIO

In Figure 8.1 the cover ratio of the repeat sales subsample is plotted over time. The cover ratio shows the proportion of all transactions covered by the repeat sales subsample. For month t the cover ratio is calculated as:

$$\text{cover ratio}_t = \frac{K_t^- + K_t^+}{2N_t} \tag{8.7}$$

where K_t^- is the number of first sales included in the repeat sales subsample for month t, K_t^+ is the number of second sales embodied in the same subsample, and N_t represents the number of transactions included in the complete NVM data set for that month. The cover ratio would be equal to one, if each transaction was both the second sale of an old repeat sales pair and the first sale of a new pair. However, as long as new houses are built and old houses are taken down, a cover ratio of one is unfeasible. For our data set an average cover ratio of 0.21 is found.

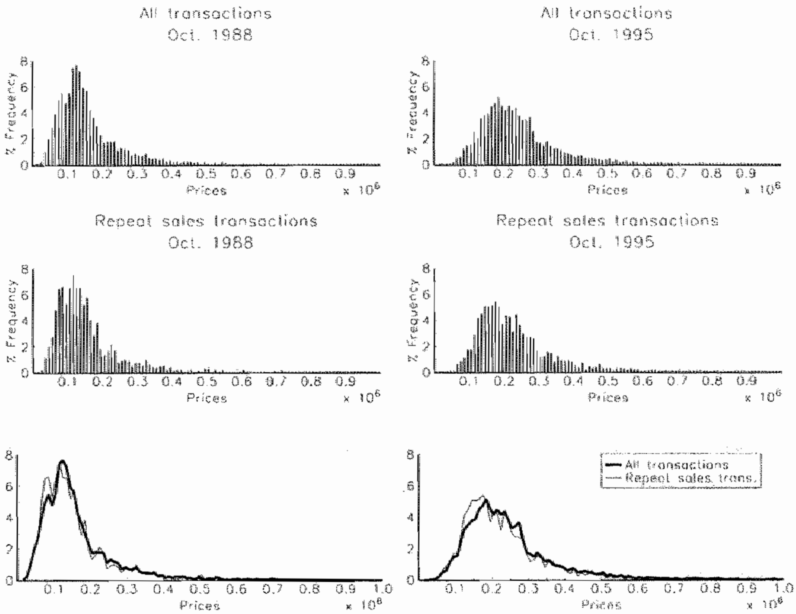

FIGURE 8.2: DISTRIBUTION OF RESIDENTIAL PROPERTY PRICES IN THE NETHERLANDS

The histograms in this figure show the relative frequency distributions of house prices in different months. The upper diagrams are based on all transactions, while the middle only use house prices included in the repeat sales subsample. The lower diagram shows the difference between both distributions. Note that prices are expressed in nominal guilders.

For both the total population and the repeat sales subsample, the relative frequency distributions of house prices are plotted in Figure 8.2 for October 1988 and October 1995, respectively. The upper diagram of this figure is based on all observed sale prices, while the middle diagram only uses house prices included in the repeat sales subsample. Both diagrams show that house prices are highly skewed to the right. For such heavily skewed distributions, the median is generally considered to be a better measure of central tendency than the mean. A property index based on the median of house prices is therefore recommended above an index based on the mean. A constant quality property index is naturally preferable to both summary statistic methods.

To compare the relative frequency distributions at one glance the relative frequency polygons of both the total population and the repeat subsample are plotted in the lower diagram of Figure 8.2. The lower diagram shows that for all three periods the house prices in the repeat sales subsample are only slightly more skewed to the right than the house prices in the total sample.

8.5 Results

There are different techniques for constructing a repeat sales index. As mentioned previously, the introduction of the original technique proposed by Bailey, Muth and Nourse (1963) was followed by the adjustments made by Case and Shiller (1987). In Figure 8.3 the Dutch residential property indices are plotted for the two alternative repeat sales estimation methodologies. Alongside these repeat sales indices, Figure 8.3 also shows two summary statistic indices, based on the mean and median of the realized sale prices.

As previously mentioned, Freddie Mac and Fannie Mae further modified the weighted repeat sales methodology in their Consumer Mortgage Home Price Index (CMHPI) for the US market. However, applying this methodology to the Dutch data results in an index which is very similar to the weighted repeat sales index as illustrated in Figure 8.3 and is therefore not plotted here.

In contrast to the repeat sales indices, the two summary statistic indices originally revealed a statistically significant seasonal pattern. Both the mean and median index shown in Figure 8.3 are corrected for seasonality by regressing the index on a set of eleven dummy variables, one for each month. This resulted in the loss of 11 months of observations. For purpose of comparison, the base period for the repeat sales indices is also moved from May 1973 to April 1974. The difference between the mean and median index is small, as is the difference between the alternative repeat sales indices. However, the dissimilarity between the mean and median indices on the one hand and the repeat sales indices on the other is remarkable. This discrepancy seems to correspond with the conclusion drawn by Gatzlaff and Haurin (1996): during periods of economic growth the repeat sales index for residential property increases faster than both summary statistic indices, while during periods of economic weakness it decreases more rapidly.

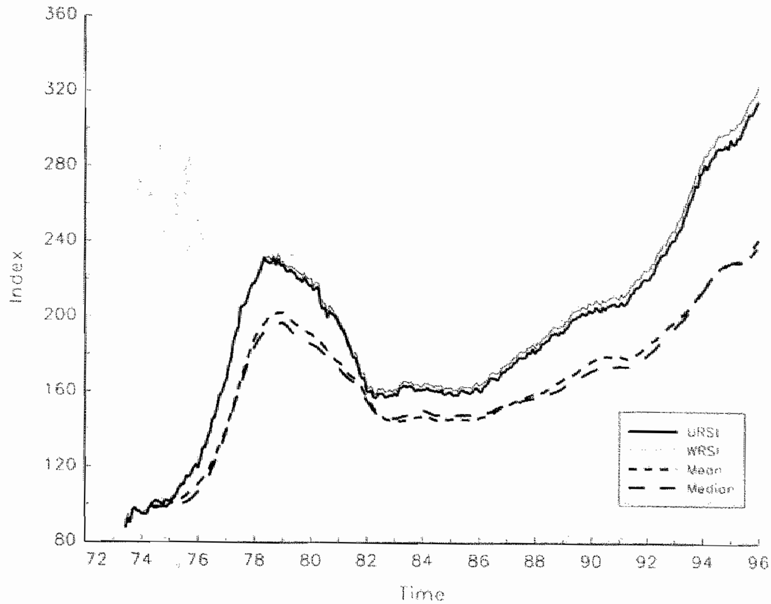

FIGURE 8.3: INDICES FOR DUTCH RESIDENTIAL PROPERTY
This figure shows alternative price indices for residential property in the Netherlands. *URSI* and *WRSI* represent the unweighted and weighted repeat sales index, respectively. *Mean* and *Median* reflect the indices based on the corresponding summary statistics.

All indices are based on observed transactions. The aversion of homeowners to sell their property during a downturn in the housing market might result in a selection bias for all four indices. Furthermore, in contradiction to the summary statistic indices which are based on the full data set of more than one million transactions, the repeat sales methodology utilized only a subsample of the observed transactions to estimate the indices. This sample selectivity might cause another bias in these indices. Following Goetzmann (1993) we studied the distributions of the separate series returns to examine this.

In Figure 8.4 the distribution of the monthly increments of both the unweighted and weighted repeat sales index are plotted. Additionally, the distributions of the cross-sectional mean and median indices are included in the figure. The summary statistics of these distributions are presented in Table 8.1. The statistics correspond with the characteristics of many financial time series; moderate autocorrelation and leptokurtosis.

A positive skewness parameter indicates that the upper tail of the distribution is thicker than the lower tail such that the distribution is skewed to the right. If it is true that people avoid selling their homes during a downturn of the market we would expect to find a right-skewed distribution of returns and therefore a positive value of the skewness measurement. As Table 8.1 illustrates the results are not unambiguous. The distribution of the increments

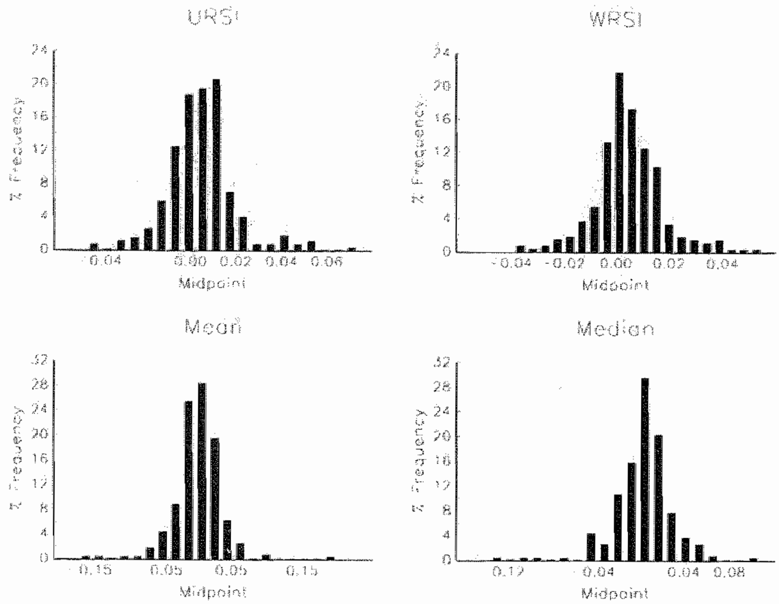

FIGURE 8.4: DISTRIBUTION OF MONTHLY NOMINAL RETURNS
This figure shows the distributions of the nominal returns based on alternative housing price indices. *URSI* and *WRSI* correspond with the unweighted and weighted repeat sales index, respectively. *Mean* and *Median* are based on the summary statistic indices.

in the repeat sales indices exhibit positive skewness. This suggests that homeowners are unwilling to sell their property during a recession period in the housing market. The negative skewness measurements found for the distribution of the returns on the summary statistic indices suggest the opposite. To study to what extent this contradiction causes the dissimilarity in the indices as plotted in Figure 8.3, a mean and median index based solely on the repeat sales subsample is constructed. The divergence between both groups of indices remains the same. This rules out the argument that a selection bias has crept into the repeat sales subsample, implying that the puzzle of the discrepancy between the repeat sales indices and summary statistic indices remains unsolved.[7]

In order to analyze how the wealth of homeowners changed over the years, these nominal indices need to be adjusted for inflation. The IMF consumer price index for the Netherlands, adjusted for seasonality, is used for this. The resulting real weighted repeat

[7] To examine whether this divergence also holds in the US we used the Case and Shiller data (1987, 1990) to construct both a weighted repeat sales and a median index. The results are similar to what we found for the Dutch market; When estimated for submarkets (the 4 cities in the US data and the 12 provinces in the Dutch data) the repeat sales and the median index follow each other closely. However, when all data are used to construct one "*umbrella*" index, the discrepancy between a weighted repeat sales and a median index stands out.

TABLE 8.1: SUMMARY STATISTICS

Method	URSI	WRSI	Mean	Median
Average increment	0.48	0.47	0.35	0.33
Standard Deviation	1.49	1.29	3.24	2.48
Skewness	0.69	0.31	-0.07	-0.67
Kurtosis	5.99	5.10	10.63	7.84

Note: The period of observation is from May 1973 to December 1995. The average increment and standard deviation are presented as monthly percentages. The skewness of a series x_t is calculated as $SK = \frac{1}{\sigma_x^3} E\left[(x_t - \bar{x})^3\right]$. The thickness of the tails is measured by the kurtosis: $KU = \frac{1}{\sigma_x^4} E\left[(x_t - \bar{x})^4\right]$.

sales index for Dutch residential property is shown in Figure 8.5 together with the 95% confidence interval. Since no seasonality correction is needed, the base month was moved back to the first month of relevant observations, that is May 1973.

Figure 8.5 clearly shows the real estate bubble of the late seventies and early eighties. This bubble was mainly caused by high inflation levels while the mortgage rate did not increase proportionally. Between January 1975 and June 1976, the inflation rate was mostly higher than the mortgage rate. This is illustrated in the lower diagram of Figure 8.6. The upper diagram of this figure shows that the inflation peaked in January 1975. The annualized inflation rate in that month was 11.09 percent while the mortgage rate was only 10.71 percent. Additionally, the interest paid on a mortgage was and still is tax deductible. This means that a homeowner who financed her house with a mortgage in January 1975 while facing an income tax of 50 percent made a profit of 5.74 percent. These "free lunches" drove up housing demand and therefore prices. Another explanation for the real estate bubble is that homeowners anticipated an increase in future incomes by encumbering a graduated payment mortgage. In March 1978 the real house prices reached a peak which has not been achieved since then. Because there was no economic foundation for real house pricing to be this high, the bubble exploded.

The bubble exploded when the inflation decreased such that the difference between the mortgage and inflation rates changed to normal proportions. Additionally, due to the economic recession in the early eighties, many mortgagors faced financial difficulties. Especially, those mortgagors whose property was encumbered by a graduated mortgage encountered financial difficulties.

Since 1986 real house prices are increasing steadily, with the exception of 1990. As Figure 8.5 shows, the real weighted repeat sales index stayed almost constant during that year. The explanation for this can be found in the tax revision which took effect on the first of a January 1990, whereby the sum of income taxes and premiums levied by the government decreased for most people. However, due to diminished tax deductibility on mortgage interest payments this revision resulted in higher net monthly mortgage costs and therefore decreased housing demand. As Figure 8.5 illustrates, this effect dominated the

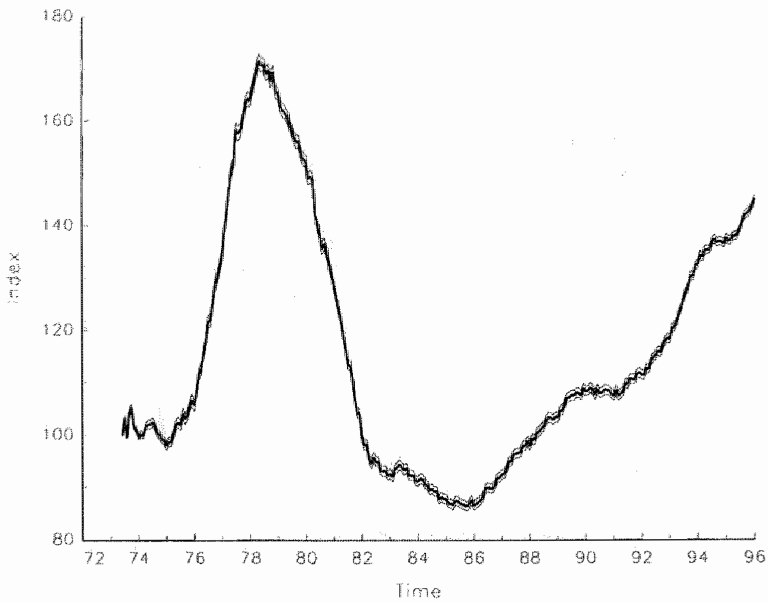

FIGURE 8.5: REAL WEIGHTED REPEAT SALES INDEX FOR THE NETHERLANDS

This figure shows the real weighted repeat sales index for Dutch residential property together with the 95% confidence interval.

income effect which stipulates that housing demand will rise if disposable income increases. At the end of 1995, the value of the real weighted repeat sales index is 144.88, which is equal to its value on March 1977 and April 1980.

8.6 Conclusion

In this paper we have constructed a repeat sales index for residential property in the Netherlands. The data used to estimate repeat sales indices were supplied by the Dutch Association of Real Estate Agents, which covers 57% of the Dutch housing market. In total, 228,144 repeat sales pairs were used to calculate alternative repeat sales indices. Alongside the unweighted repeat sales index, a weighted repeat sales index (WRSI) was constructed by applying a Generalized Least Squares estimation technique to correct for heteroskedasticity in the index. We find no notable differences in the estimated repeat sales indices. When compared with indices based on cross-sectional means or medians our WRSIs shows a stronger growth. This difference remains even when the same repeat sales sample of houses is used to compute both summary statistic indices. Apart from the difference in levels, the repeat sales index also has much lower volatility when compared

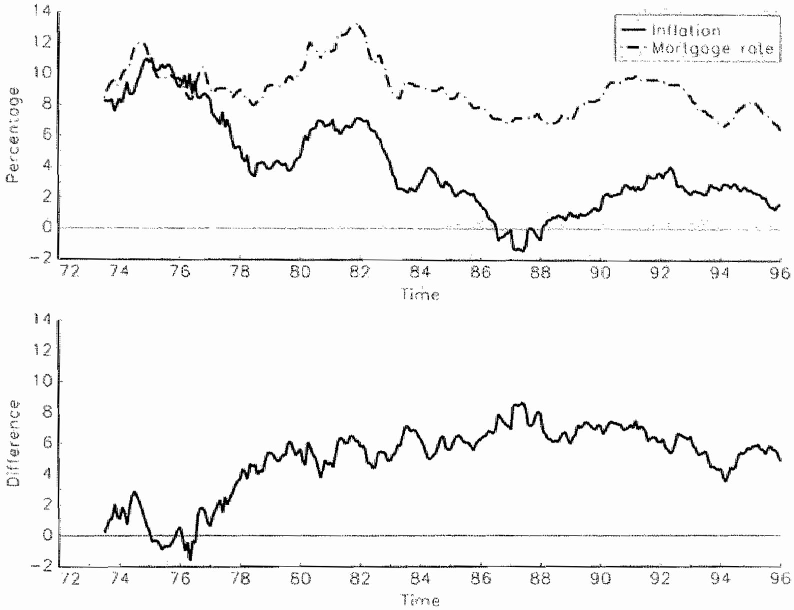

FIGURE 8.6: DUTCH INFLATION AND MORTGAGE RATES

The upper diagram of this figure shows the time series of the inflation rate and the mortgage rate. The lower diagram plots the difference.

with the mean and median indices. This is a consequence of the methodology itself. Nevertheless we are confident of the validity of the repeat sales index as a representative indicator of the value of residential properties in the Netherlands.

The estimated index clearly shows the real estate bubble of the late seventies and early eighties. The basis of the nominal index is May 1973. Five years later a level of 255.37 was reached, such that the average annual return during that period exceeded 20%. After this period of sharply rising prices the real estate bubble exploded. Over the following 7 years and 3 months the nominal index decreased by an average of 4.74% per annum. In September 1985 the housing prices started to rise again. Between September 1985 and December 1995 the index increased by an average of 7% a year. For comparison, the average annual stock return for each of these three periods was equal to 2.31, 18.61 and 12.02%, respectively. These figures include dividends but the return on residential properties is only based on price fluctuations, and the rent-savings are ignored. For first-time buyers, such savings are undoubtedly a major factor in choosing to purchase a house rather than renting one, especially when roughly the same monthly payment could result in full ownership of the house. Homeowners who hope to save towards their pension in this way do face the risk of relatively low house prices at the time they want to convert their property into money. These individuals should be particularly interested in the risk and return characteristics of residential property.

Chapter 9

Empirical mortgage prepayment behavior in the Netherlands

9.1 Introduction

The mortgage valuation models described in Chapters 5 and 7 assume that prepayments are solely interest rate driven. American prepayment data, however, show that residential mortgage borrowers regularly prepay their mortgage even though the refinancing rate exceeds the contract rate on the existing mortgage, while others fail to exercise their prepayment option when it would financially be wise to do so. Optimal call valuation models can not explain this behavior. Valuation models in which prepayments are exogenously specified override this shortcoming. However, empirical models have other limitations. For example, Lucas (1987) pointed out that relations found in empirical studies are based on historical observations and can therefore not be used to consider the future impact of today's policy changes. In other words, it is difficult to predict prepayment behavior if contracts are introduced with different conditions.

Exogenous prepayment valuation models can be divided into two categories. Mortgage prepayment models developed by Dunn and McConnell (1981a,b) , Brennan and Schwartz (1985), Kau, Keenan, Muller and Epperson (1992), Giliberto and Ling (1993), Archer and Ling (1993) and Stanton (1995) take an optimal call valuation model as a starting point and add exogenous calls which are unrelated to interest rates. On the other hand, there are strictly empirical prepayment models, which do not assume any optimal behavior. The prepayments are solely based on historical relations with explanatory variables. This second category of exogenous prepayment models includes the proportional hazard models by Green and Shoven (1986) and Schwartz and Torous (1989), and the more descriptive models by Richard and Roll (1989), Kang and Zenios (1992) and Golub and Pohlman (1994).[1]

The purpose of this chapter is to contribute to the understanding of mortgage prepay-

[1] Chapter 3 discusses these models in more detail.

ment behavior in the Netherlands. For this we analyze the prepayment data of a large mortgage provider in the late eighties and early nineties. The observed prepayment rates are related to various measurable factors suggested by economic theory. Section 9.2 describes the data used in this chapter. Unfortunately, our data set has serious limitations: the number of included contracts is rather small (as is the number of observed prepayments) and the sample period is limited to the first five and a half years of a contract.

The remainder of this chapter is organized as follows. Section 9.3 compares the observed prepayment activity with optimal prepayment behavior. In Section 9.4 the prepayment data are modelled on the basis of five factors affecting prepayment behavior: seasonality, refinance incentives, aging, loan-to-value and housing price effects. In Section 9.5, these five factors are considered simultaneously in a multi-variate analysis. Section 9.6 concludes.

9.2 Description of the data

9.2.1 Individual loan data

The mortgage data analyzed in this chapter are from clients of *Assurantieconcern Stad Rotterdam anno 1720 N.V.*, a large independent Dutch insurance company quoted on the Amsterdam Stock Exchange. For a period of almost five years, 183 traditional mortgages with life insurance and 150 savings mortgages were followed.[2]

Since almost no repayments take place during the term of these mortgage contracts, the outstanding balances remain untouched. Consequently, the interest costs to the mortgagors do not decline over the years. However, interest paid on a mortgage is tax deductible in the Netherlands, while the return on the life insurance of both the traditional and savings mortgage is not taxed as long as some fiscal conditions are met. These restrictions are described in Chapter 2. An individual who uses a traditional mortgage with life insurance or a savings mortgage to finance his home makes optimal use of the facilities offered by the tax authorities. These tax facilities make (partial) prepayment of mortgages with life insurance and savings mortgages often unattractive. Therefore we expect to observe much less prepayment activity in our Dutch data set than in most American studies. Table 9.1 confirms this expectation. This table separates the analyzed contracts by their month of issue, April, May and June 1990, respectively. These months were chosen because *Assurantieconcern Stad Rotterdam anno 1720 N.V.* issued their first savings mortgage in April 1990. The prepayment behavior of the considered contracts is followed until December 1995. In the underlying time span 91 partial prepayments and 46 complete prepayments on mortgages with life insurance occurred. For the savings mortgages these numbers are 47 and 14, respectively.

Regularly, individuals buy a new home before the old one is sold. Often the new home is financed by a mortgage with a bridging loan. Commonly, this bridging loan has a maturity

[2] For a more detailed discussion on both mortgage types see Chapter 2.

TABLE 9.1: SUMMARY STATISTICS REGARDING MORTGAGE CONTRACTS

Mortgage	Traditional			Savings		
Month of issue	April	May	June	April	May	June
Contracts	67	74	42	50	69	31
Partial prepayments	35	34	22	10	23	14
Complete prepayments	14	15	17	0	10	4

This table reports the number of traditional mortgages with life insurance and savings mortgages included in the sample. All contracts are issued in the second quarter of 1990. The second row displays the month of issue. The third row reports the number of contracts issued in the corresponding month. Row four states the number of partial prepayments observed between the month of issue and December 1995. Row five presents the number of complete prepayments observed during that period.

of 12 months or less, and will be repaid with the revenue of the old house once it is sold. The contracts studied in this chapter are corrected for these bridging loans. The analyzed contracts are also cleaned of loans to employees and related insurance brokers who receive a discount on the contract rate and thus exhibit different prepayment behavior.

TABLE 9.2: SUMMARY STATISTICS REGARDING CONTRACT RATES

Mortgage	Traditional			Savings		
Month of issue	April	May	June	April	May	June
Average	8.26	8.30	8.67	8.88	9.01	9.00
Stand. Dev.	0.54	0.57	0.53	0.56	0.60	0.58
Maximum	9.20	9.60	9.25	9.95	9.85	9.75
Minimum	7.00	6.80	6.85	7.30	7.25	7.40
Median	8.25	8.32	8.95	8.95	9.25	8.95

This table reports the initial contract rates of the mortgages issued by *Assurantieconcern Stad Rotterdam anno 1720 N.V.* in the second quarter of 1990.

Figure 9.1 illustrates the distribution of the initial contract rates for which the summary statistics are reported in Table 9.2. Both histograms in Figure 9.1 are based on a mixture of contracts. Both first and second mortgages are included, as are contracts whose mortgage rate is fixed for a 1, 5 or 10 year period, the time to maturity varies between 20 and 30 years and also the loan-to-value ratio differs substantially between the individual contracts.[3] The diversity of contract specifications is reflected in the high dispersion of the contract rates as illustrated by both Figure 9.1 and Table 9.2. The contracts have in common that only 10% of the initial loan can be prepaid in a calendar year without a penalty.

[3] The loan-to-value ratio compares the amount of the loan with the value of the underlying property.

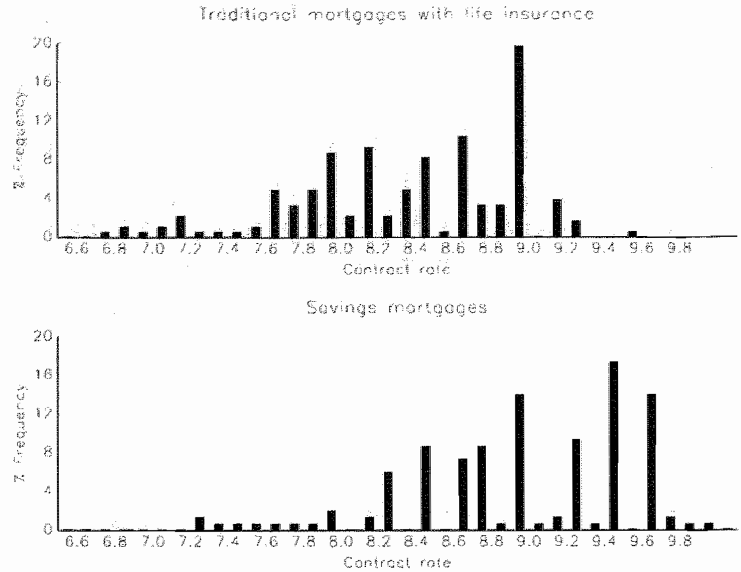

FIGURE 9.1: INITIAL CONTRACT RATE

The histograms in this figure show the frequency distributions of the initial contract rates of the mortgage contracts studied in this chapter.

9.2.2 Aggregate data

We also have prepayment data at an aggregate level which refers to the total amount of prepayments received by *Assurantieconcern Stad Rotterdam anno 1720 N.V.* in each month between January 1986 and June 1994. The book value of the outstanding mortgage portfolio at the beginning of each month is also known, so that the prepayment rates can be calculated and the monthly fluctuations be analyzed. A distinction is made between traditional mortgages with life insurance and savings mortgages. The first savings mortgage was issued only in April 1990, such that prepayment data for this loan type is only available from May 1990 on. The contract rates of the mortgages which prepay are known. However, the remaining time to maturity, the fixed-rate period, the initial loan and the underlying value of the property are unknown.

9.3 Observed and interest rate driven prepayment behavior

Chapters 5 and 7 consider optimal prepayment behavior where the borrower replaces the mortgage if this reduces his future monthly payments. In Figure 9.2 this optimal prepayment behavior is compared with the prepayments observed in the Stad Rotterdam data set concerning the individual contracts.

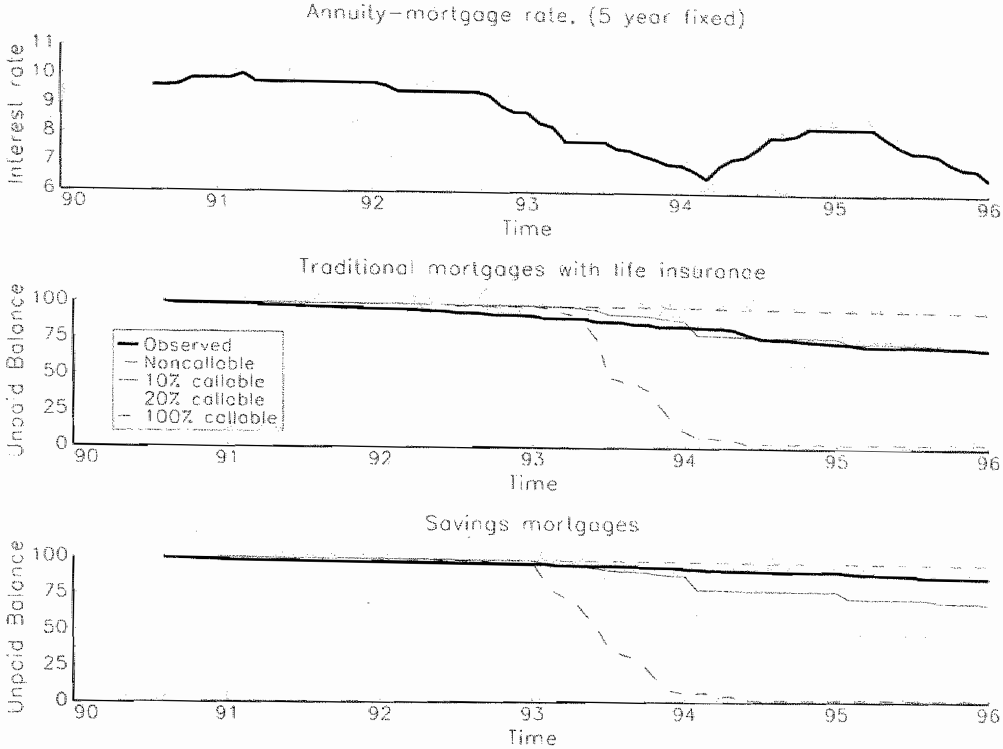

FIGURE 9.2: OBSERVED VERSUS OPTIMAL PREPAYMENT BEHAVIOR

The upper diagram of this figure shows the time series of the annuity-mortgage rate. The other two diagrams compare the observed prepayment behavior with the optimal prepayment behavior.

The upper diagram illustrates the mortgage rate on an average Dutch annuity-mortgage contract whose maturity is 30 years with the contract rate being fixed for a five year period. The contract rate of comparable traditional mortgages with life insurance is equal to this annuity rate, while a practitioners rule-of-thumb states that the contract rate of savings mortgages is 30 basis points higher. Throughout this chapter, this rule-of-thumb is assumed to hold. To adjust for alternative fixed-rate periods, we used spreads observed on the capital market. The spread between the Holland Interbank one year yield and the DataStream constructed benchmark for redemption yields on Dutch government bonds with a maturity of five years, and the spread between this latter variable and the comparable 10 year benchmark, were added to the five year fixed mortgage rates. This way time-varying spreads were used to approximate time series of mortgage rates for contracts whose rate is fixed for a one and ten year period, respectively.

These time series are utilized to model prepayment behavior as was done in Chapters 5 and 7. Each month the monthly payment M_t is calculated as if a similar new contract starts with a maturity equal to the remaining time to maturity of the existing contract, and a contract rate which is equal to the mortgage rate corresponding to that month. The principal of this new contract is equal to the outstanding loan multiplied by $(1+c)$, where c are the up-front costs a borrower has to pay to replace the old contract by a new one. If the resulting monthly payment M_t is less than the original M_0 and the annual prepayment limit has not been reached, the borrower is assumed to prepay (part of) the mortgage. Hereby we assume that prepayments never exceed the annual penalty-free restrictions, that the up-front costs equal one percent of the outstanding balance and that existing contracts are only replaced by new contracts with the same fixed-rate period and by contracts of the same type, either traditional mortgages with life insurance or savings mortgages.

The middle diagram of Figure 9.2 illustrates the consolidated outstanding balance (COB) of the pool of traditional mortgages with life insurance.[4] The bottom diagram plots the same for savings mortgages. The unpaid balance was rescaled such that the consolidated outstanding balance on July 1^{st}, 1990 corresponds with one hundred. The bold line displays the observed unpaid balance at different points in time. The thin lines illustrate what the unpaid balance would have been if the above described prepayment decision rule was followed, whereby alternative annual prepayment restrictions are considered. Both the observed and the optimal prepayment behavior include periodical repayments.

Figure 9.2 shows that the redemption of traditional mortgages with life insurance is larger than the redemption of savings mortgages. This is caused by both higher periodical *re-* and *pre*payments. In the second half of the sample period, the graph of the unpaid balance of the traditional mortgages with life insurance shows much resemblance with the line representing the unpaid balance of optimal-called annuity-contracts with an annual 10 percent prepayment limit. Since the prepayment option on a traditional mortgage with life insurance is also limited to 10 percent per calendar year, this result suggest rational prepayment behavior of the mortgagor.

The lines representing the partially callable contracts show a decline in the unpaid balance at the beginning of 1995, despite the fact that the interest rate does not decrease. This is purely a calendar effect: Given the contract rates occurring at the end of 1994, borrowers want to prepay their loans. However, if their annual prepayment limits have already been reached, they are not allowed to do so. Together with the new year 1995, a new prepayment option comes along which gives the mortgagor the right to prepay an additional 10 or 20 percent of the initial loan during that calendar year.

[4] The consolidated outstanding balance (COB) at time t is equal to the outstanding loan at that time minus the money accrued in the savings account. Data regarding the savings accounts are not available. However, the contract rate is known so that the amount included in the savings account can be estimated.

9.4 Modelling Dutch prepayment data

Chapter 3 discusses four explanatory variables which appear in different qualities in different empirical mortgage prepayment studies. The *refinancing incentive* reflects that home-owners tend to refinance their mortgage when the current mortgage rate is far enough under the contract rate. *Seasonality* measures the correlation between prepayment rates and the month of the year. The phenomenon that prepayment rates are low in the beginning of the contract and increase towards a stable level as the mortgage ages is known as *seasoning*. And finally, *burnout* describes the decline in prepayment rates as mortgage contracts age through interest rate cycles. Both seasoning and burnout are aging factors. The first one describes the prepayment behavior at the beginning of the mortgage contract, while the latter one focuses on later stages of the contract life. Other economic variables are also sometimes included, *e.g.* loan-to-value ratios and housing price developments.

The various prepayment determinants are discussed in this section. The discussion borrows from Richard and Roll (1989), Kang and Zenios (1992) and Golub and Pohlman (1994). Where possible our results will be compared with theirs.

9.4.1 Seasonality effects

Seasonality captures the cyclical variations in prepayment rates. The seasonal pattern is expected to be related to the activity on the housing market. It is commonly believed that residential property transactions increase in the spring and gradually reach a peak during summer months. This increased activity is expected to result in a similar seasonal pattern of the prepayment rates. This is empirically confirmed in studies by Richard and Roll (1989) and by Kang and Zenios (1992); Prepayment activity is at its lowest in March, after which it increases to more moderate levels in spring and summer, but the real peak is reached in autumn or early winter.

Figure 9.3 illustrates the seasonality effects in the Netherlands based on the aggregate data set. The prepayment rate is calculated by dividing the received prepayments by the consolidated outstanding balance at the beginning of the month. The ratio is multiplied by 100 and represents the prepayment per 100 guilders principal outstanding. The upper diagram illustrates the prepayment rate over the years for both contract types. The lower left diagram plots the average prepayment rate per month of traditional mortgages with life insurance. The seasonal fluctuations show much resemblance with the patterns found by Richard and Roll (1989) and Kang and Zenios (1992). The prepayment rates are relatively low in the beginning of the year, begin to rise in the spring and reach a peak in July which suggests that the delay in passing through prepayments is rather small in the Netherlands.

The annual prepayment restrictions embodied in Dutch mortgages and the fact that a calendar year coincides with a tax year might cause the other prepayment peak in December.

The bottom right diagram of Figure 9.3 displays the seasonal pattern of savings mort-

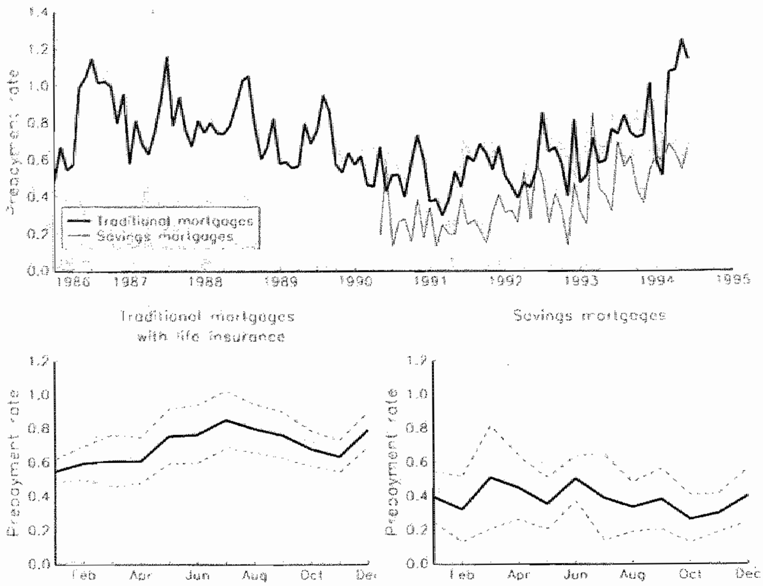

FIGURE 9.3: SEASONALITY EFFECT

The upper diagram of this figure shows the prepayment rate over the years. The lower diagrams plot the average prepayment rate per month. The thin-dotted lines display the 95% confidence intervals.

gages' prepayment rates. This time the seasonal pattern is less clear.

9.4.2 Refinance incentives

Declining interest rates provide an incentive for homeowners to refinance their mortgage. Following Richard and Roll (1989), Kang and Zenios (1992), and Golub and Pohlman (1994), we measure the refinance incentive by computing the annuity value per guilder of principal outstanding. The present value of a 30 year annuity in month t, A_t, per guilder of monthly payment equals:

$$A_t = \frac{1 - (1 - y_t)^{t-360}}{y_t}, \tag{9.1}$$

where y_t is the mortgage rate in month t.[5] When y_0 represents the contract rate of the existing mortgage, the outstanding balance, per unit of monthly payment, can be written as:

$$OB_t = \frac{1 - (1 - y_0)^{t-360}}{y_0}. \tag{9.2}$$

Hence, the annuity value per guilder of principal outstanding reads:

[5] Actually, this underestimates the value of the annuity since the mortgage rate is determined for partially callable mortgage loans.

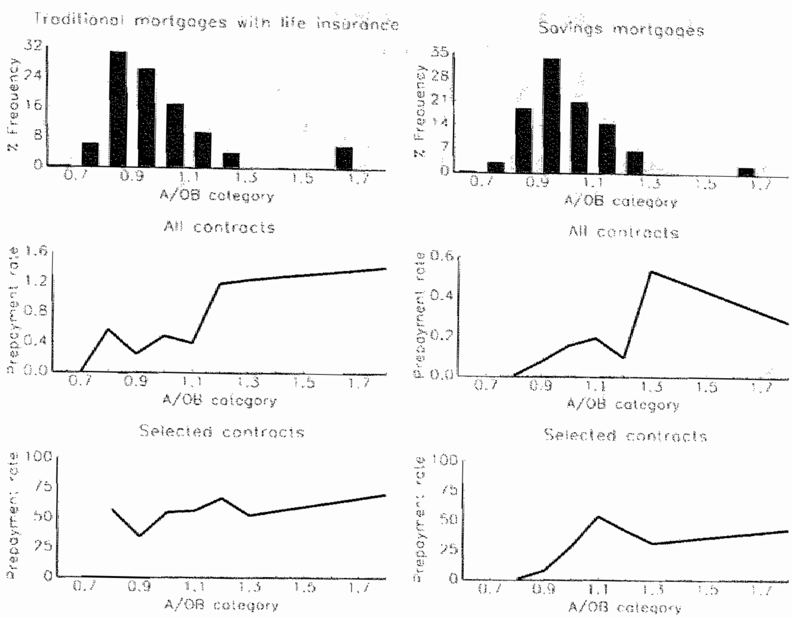

FIGURE 9.4: REFINANCE INCENTIVE

The upper diagrams of Figure 9.4 show the frequency distribution of the $\frac{A_t}{OB_t}$ ratio categories for both mortgage types. The middle and lower diagrams plot the weighted average prepayment rate per category.

$$\frac{A_t}{OB_t} = \frac{y_0}{y_t}\left(\frac{1-(1-y_t)^{t-360}}{1-(1-y_0)^{t-360}}\right). \tag{9.3}$$

Borrowers will prepay their mortgage when the $\frac{A_t}{OB_t}$ ratio exceeds some critical value. The exact level of the ratio that triggers prepayment depends on the refinancing costs, which vary among individuals, and the stochastic process for interest rates.

To incorporate a potential delay in responding to a change in interest rates, Richard and Roll (1989), Kang and Zenios (1992) and Golub and Pohlman (1994) use weighted averages of recent values of the ratio to measure the refinance incentive. Their results illustrate a non-linear relation between the prepayment rate and the refinance incentive ratio. For ratios below one, the refinance rate exceeds the contract rate and prepayment activity is relatively low. The prepayment rate increases rapidly for refinance incentive ratios larger than 1.

Figure 9.4 summarizes the results for the Dutch data set concerning the individual contracts. To determine the refinance incentive for each contract in each month, the average of its three most recent $\frac{A_t}{OB_t}$ ratios is used. The resulting ratios are divided over 9 categories. The first category contains ratios less than 0.6, the other ones have a bandwidth of 0.10, while the last one embodies ratios exceeding 1.30. The upper diagrams of Figure 9.4 show the frequency distributions of these ratio categories for both mortgage types. For each category, the weighted average prepayment rate is calculated and plotted in both middle diagrams.[6] In conformity with Figure 9.3, this prepayment rate represents the prepayment per 100 guilders principal outstanding. Even though these middle diagrams suggest that the prepayment activity increases with $\frac{A_t}{OB_t}$ values, there is no clear S-shape, non-linear relation visible. The relation is distorted by a few outliers which have a major effect on the outcome, and by the fact that most of the time no prepayments are observed. If these zero-prepayments are left out, we find a pattern as illustrated by the lower diagrams of Figure 9.4.

TABLE 9.3: REFINANCE INCENTIVE

$$pr_t = \alpha + \beta_1(\frac{A_t}{OB_t}) + \beta_2(\frac{A_{t-1}}{OB_{t-1}}) + \beta_3(\frac{A_{t-2}}{OB_{t-2}}) + \beta_4(\frac{A_{t-3}}{OB_{t-3}}) + \beta_5(\frac{A_{t-4}}{OB_{t-4}}) + \epsilon_t$$

	α	β_1	β_2	β_3	β_4	β_5	R^2
value	-1.16	-4.59	0.42	7.86	0.60	-2.69	0.32
standard error	0.47	2.25	3.27	3.39	3.40	2.34	

Period: Nov. 1990 - Dec. 1995, hence T = 62, pr_t represents the prepayment rate at time t, A_t is the annuity value per guilder of principal outstanding at time t, and OB_t is the outstanding balance per unit of monthly payment.

In Table 9.3 we regress the prepayment ratio on the prevailing refinance incentive ratio and four of its lags in months. During these months the mortgagor can reconsider the new information, compare new offers and finally make a decision to refinance or not. The R^2 resulting from the regression shows that 32 percent of the variability in prepayment rates can be explained by current and past refinance incentives. Not all parameters are positive due to multicollinearity. However, the sum of the parameters is equal to 1.61, which indicates that the prepayment activity is positive related to refinance incentives.

9.4.3 Aging

Both seasoning and burnout refer to the aging of the mortgage loan. Seasoning describes the general idea behind the PSA prepayment benchmark, discussed in Chapter 3. It reflects that prepayment rates are low shortly after the mortgage is originated and increase to a stable level in course of time. Burnout on the other hand tries to measure the tendency of prepayment rates to slow down over time.

Recall that the American PSA benchmark prescribes prepayment rates to increase linearly for the first thirty months of the contract's existence. After these initial thirty months the monthly prepayment rates are assumed to be 6% a year of the outstanding

[6] Weighted averages are calculated based on the consolidated outstanding balance (COB) of each mortgage contract.

balance for the remainder of the mortgage term. To analyze the seasoning effect, Richard and Roll (1989) and Kang and Zenios (1992) distinguish alternative mortgage pools based on the refinance incentive ratio. Richard and Roll find that discount mortgages ($\frac{A_t}{OB_t} = 0.8$) fully level off in about nine years, although 75% seasoned after five years. Par mortgages ($\frac{A_t}{OB_t} = 1$) take about five years to fully season, and premium mortgages ($\frac{A_t}{OB_t} = 1.2$) need just over thirty months. Kang and Zenios find very similar results.

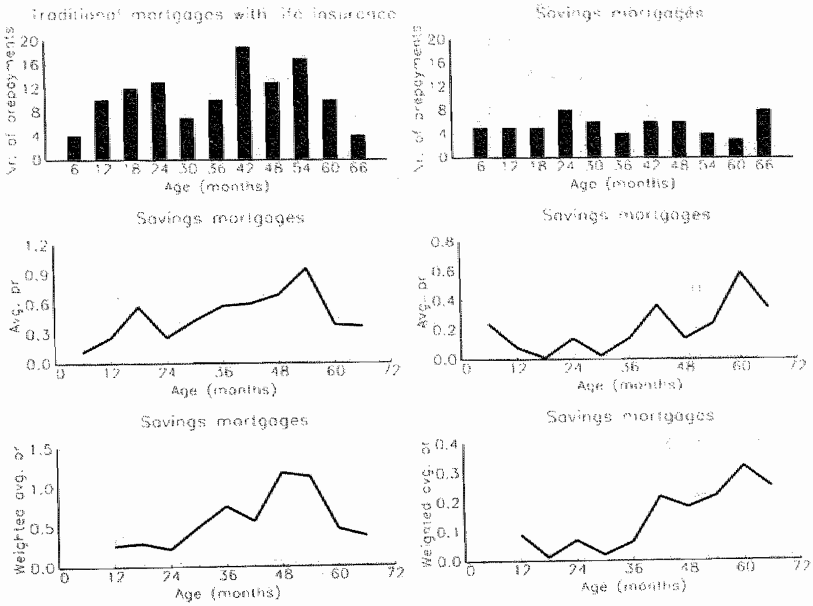

FIGURE 9.5: AGING EFFECT

The upper diagrams of this figure show the number of prepayments observed in the considered period as a function of the age of the contract. The middle diagrams plot the relation between the age of the contract and the unweighted average prepayment rate, while for the lower diagrams the rates were weighted before averaged.

Due to our limited data set, no distinction can be made between discount, par and premium mortgages in the Netherlands, nor can the age effect be studied on a monthly basis, instead a six months period is chosen. The limitations of our data set are clearly laid open in the upper diagrams of Figure 9.5. These two histograms show the number of contracts which were (partially) prepaid at a specific age. Both histograms illustrate that the prepayment frequency is rather low.

The middle diagrams display the relation between the age of a contract and the unweighted average prepayment rate, while for the bottom diagrams the prepayment rates were weighted before averaged. All suggest the same: prepayment rates gradually increase with the collapse of time. However, nothing points to a stable, seasoned prepayment level reached in the first 5 to 6 years of a contract's existence.

9.4.4 Loan-to-value

Loan-to-value is the ratio of the initial loan to the forced-sale value of the underlying residential property. The higher this ratio, the smaller the lender's protection for losses which might occur when the borrower defaults and the property has to be sold. As such, the loan-to-value ratio is more closely related to default risk than to prepayment risk. However, in our data set we can not make a distinction between the termination of a contract due to prepayment or default.

Before turning to the loan-to-value analysis, the upper diagram of Figure 9.6 summarizes the relative frequency distributions of the property values underlying the individual mortgage contracts analyzed in this chapter. In keeping with Chapter 8, the distributions of house values are skewed to the right. Subsequently, Figure 9.6 illustrates that neither a relation exists between contract rates and forced-sale values of underlying homes, nor between contract rates and loan-to-value ratios. This insinuates that the reimbursement required by lenders for facing additional prepayment/termination risk is rather low. The lower diagrams indicate that a large risk premium can also not be justified. In these diagrams the number of prepayments divided by the number of contracts in a specific loan-to-value category is plotted against these loan-to-value classes.

9.4.5 Housing prices

With strong housing market conditions as observed in the considered period, borrowers who can not meet their financial burden will sell their property and prepay the mortgage. Exercising their default option is most likely much more expensive, so that this option will only be exercised when the mortgagors' equity in their homes become negative. Consequently, the number of defaults is rather low in this sample period of rising housing prices. This might also explain why no positive relation is found between prepayment activity and loan-to-value ratios.

Figure 9.7 plots the prepayment rate against the monthly mutation in housing prices. These mutations are based on the weighted repeat sales index for residential property in the Netherlands, as developed in Chapter 8. The prepayment rates are derived from the data set which contains prepayment information on an aggregate level. The scatter diagram shows no relation between the economic variables. Including lags in the housing price fluctuations does not improve this picture, nor does averaging over a longer period than one month.

9.5 Multi-variate analysis

Following the analyses of the separate effects, Richard and Roll (1989), Kang and Zenios (1992) and Golub and Pohlman (1994) integrate the factors into a multiplicative model to determine the conditional prepayment rates (CPR):

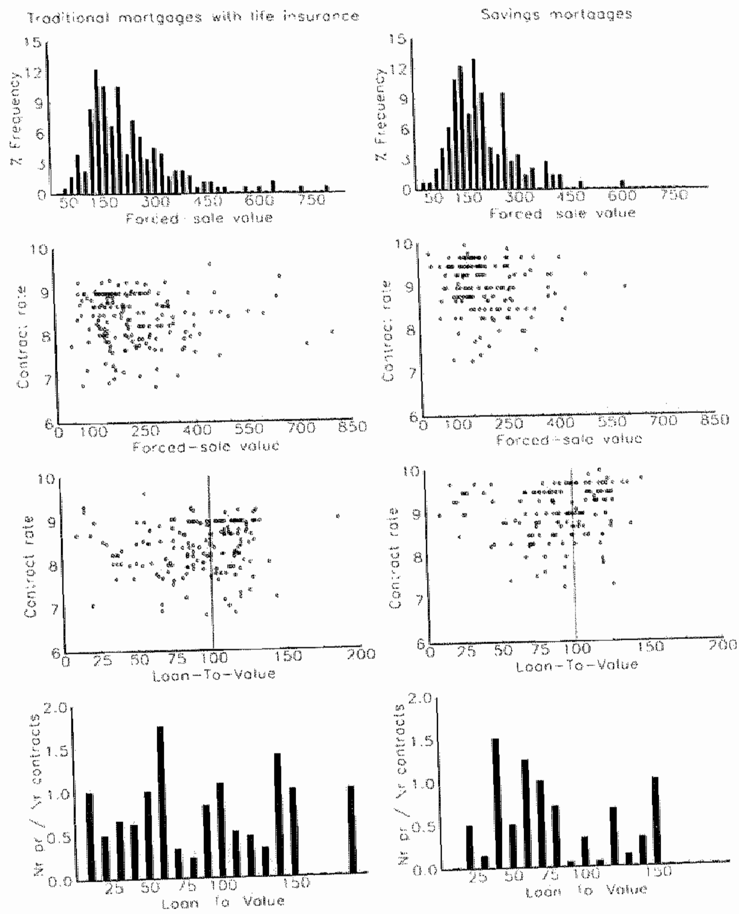

FIGURE 9.6: LOAN-TO-VALUE

The upper diagrams in this figure show the relative frequency distribution of the forced-sale value of the property underlying the mortgages. The middle diagrams relate the contract rate to the forced-sale value and the Loan-To-Value (LTV) ratio, respectively. The lower diagrams plot the number of prepayments divided by the number of contracts as a function of the Loan-To-Value ratio.

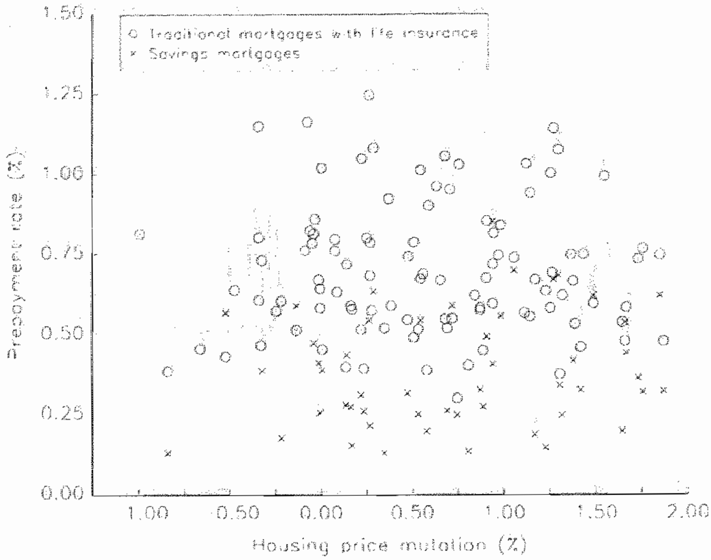

FIGURE 9.7: HOUSING PRICE EFFECT

This figure studies the relation between housing prices and prepayment activity.

$$\text{CPR} = \text{seasonality} \times \text{refinancing incentive} \times \text{seasoning} \times \text{burnout.} \tag{9.4}$$

Since not all factors were found to be significant, we have chosen for an alternative approach to combine the independent variables into one prepayment model. Rather than determining the parameters for each component separately, we estimate them simultaneously with multiple regressions. For this we have combined all individual loans into one pool. The consolidated outstanding balances of the individual contracts are used to calculate the weighted average contract rate, age, loan-to-value ratio and refinance incentives of this pool.

We started with a general multiple regression in which the prepayment rates are related to all explanatory variables described in this chapter. For the refinance incentive we used the current ratio, as well as four of its lags. The age of the loan, the loan-to-value ratio, the housing price fluctuations and dummy variables for the seasonality effect are also included in this general regression. And to capture the impact of past prepayment behavior on current prepayment rates, we added the cumulative past prepayments to this list of independent variables.

The success of the regression in predicting the prepayment rates within the sample is graphically shown in the top diagram of Figure 9.8. The goodness of fit is measured by R^2, which for this regression is equal to 53.2%. This means that more than half of the variability in the prepayment rates can be explained by the aforementioned set of explanatory variables. However, not all regression coefficients are statistically significant. Whether or not the corresponding variables should be included in the set of explanatory variables is therefore open to question. Including these variables will increase R^2, regardless the importance of the variables. We can use the *adjusted* R^2 as an alternative goodness of fit measure. This R^2_{adj} allows for the trade-off between increased R^2 and decreased degrees of freedom, such that adding an extra independent variable does not automatically lead to an increase in the goodness of fit benchmark.[7] The adjusted R^2 of the general multiple regression is equal to 30.3%. Omitting the statistically not significant explanatory variables leads to a more tractable model with an R^2_{adj} of 36.3%. This restricted regression model is summarized in Table 9.4, and the fitted values are plotted in the bottom diagram of Figure 9.8.

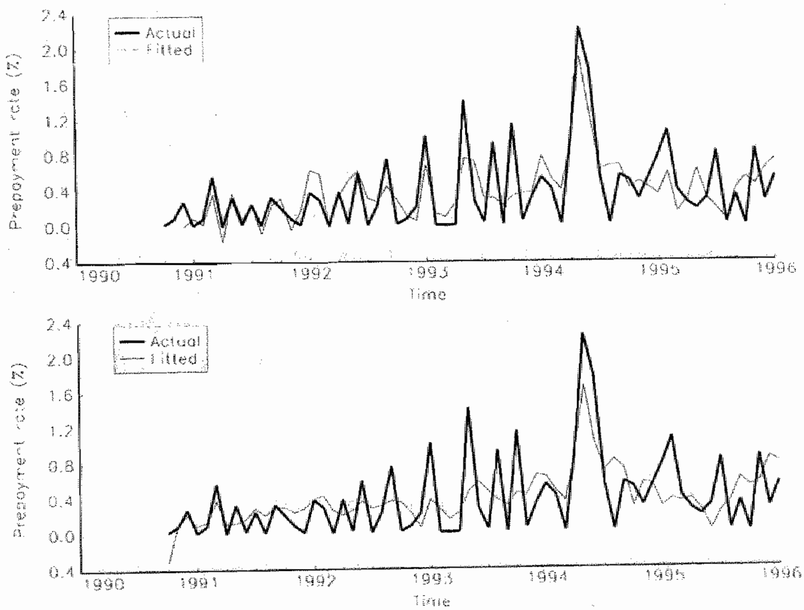

FIGURE 9.8: ACTUAL AND ESTIMATED PREPAYMENT RATES

The top diagram of this figure plots the actual prepayment rates of the constructed mortgage pool together with the fitted rates estimated by the general multiple regression. The bottom diagram shows the estimates based on the restricted multiple regression.

The seasonal dummies, the housing price fluctuations and the refinance incentives with lags of one, three and four months are no longer included in the restricted regression model.

[7] $R^2_{adj} = 1 - \frac{T-1}{T-k}(1 - R^2)$, where T is the number of observations and k is the number of explanatory variables.

$\frac{A_{t-2}}{OB_{t-2}}) + \beta_3 Age + \beta_4 LTV + \beta_5 \mathrm{CUM}(pr_{t-1}) + \epsilon_t$

β_2	β_3	β_4	β_5	R^2	R^2_{adj}
9.07	0.14	0.11	-0.07	0.41	0.36
1.83	0.05	0.04	0.03		

1995, hence $T = 64$, pr_t represents the prepayment rate
ty value per guilder of principal outstanding at time t,
alance per unit of monthly payment, *Age* is the age of
TV represents the loan-to-value ratio and CUM stands

les to the explanatory power of the model was too small. revailing refinance ratio, the two month lagged refinance he loan-to-value ratio and the prepayment history. This by the factor $\mathrm{CUM}(pr_{t-1})$ which represents the cumulative nonth. The greater past prepayments, the smaller current 0. The opposite holds for the loan-to-value ratio and the mortgage contracts only during the first five and half years $_3$ is therefore in line with the seasoning effect which states shortly after the mortgage is originated and increase to a The sample period is too short to detect a seasoned level, ayment rates increase during the first five and half years of

re included in the regression models to capture the interest he restricted regression model we suffice with the current of two months. The other lagged refinance incentive ratios reducing the explanatory power of the regression model. egression reported in Table 9.3 with its counterpart of the hows the explanatory power of the refinance incentives and erest rate driven prepayments. However, this result must ecause the strong relation between the prepayment activity arily based on the enormous prepayment peak observed in o months after the mortgage rate reached its lowest level in per diagram of Figure 9.2 displays that at the end of 1995 at the same level as in the first quarter of 1994. Despite nt activity does not approach the levels of early 1994. This henomenon: prepayments depend not only on the refinance n the path taken by the mortgage rate. Borrowers sensitive ill prepay as soon as the refinancing incentive is large enough, tgagors in the pool. Consequently, a smaller proportion of the remaining mortgagors will respond the second time the refinance incentive ratio has this value.

To capture individual incentives to prepay or default the mortgage contract, cross-sectional information should be included in the above analysis. Explanatory variables included in various cross-sectional studies are summarized by Cunningham and Capone (1990) who categorize them as mortgage related, borrower related, property related, or macroeconomic variables.[8]

Unfortunately, our Dutch data set does not allow an in-depth cross-sectional analysis which incorporates the above variables. The mortgagors' characteristics in particular are lacking. Applying a cross-sectional analysis based on the explanatory variables described in Section 9.4 will yield very similar results as our multiple regressions, because all mortgages included in our data set are issued in the second quarter of 1990. This means that they are all issued with similar conditions and contract rates, that they all have the same age and face the same trend in housing prices. This latter is also reflected in similar developments of the individual loan-to-value ratios. In other words, there is not enough variety amongst the individual contracts to distinguish one factor from another more clearly by using cross-sectional techniques rather than time series analyses.

9.6 Conclusion

This chapter provides some preliminary empirical results on mortgage prepayment behavior in the Netherlands. The prepayment data analyzed in this chapter is provided by *Assurantieconcern Stad Rotterdam anno 1720 N.V.*. The observed prepayment behavior of mortgagors whose annual prepayment option is limited to 10% of the initial loan shows much resemblance to the behavior described by the prepayment decision rule described in Chapters 5 and 7.

In this chapter we try to reveal empirical relations between the observed prepayment rates and economic variables. The analyses incorporate the main determinants of prepayment models described in the economic literature: seasonality, interest rates, age, loan-to-value ratios and housing prices.

A seasonal pattern is observed in the Dutch data set: In early spring the prepayment rates are relatively low. After this the prepayment activity increases and reaches a maximum in July. Due to the annual limited prepayment option embodied in most Dutch contracts and due to tax effects, another peak is uncovered in December. Following Richard and Roll (1989) and Kang and Zenios (1992) the refinance incentive is measured by computing the annuity value per guilder of principal outstanding. For ratios less than one, the corresponding prepayment activities are low. Only once the ratio exceeds one, the prepayment rates start to increase rapidly. Regarding the age of the contract, the results

[8] Mortgage prepayment and default studies based on a cross-sectional analyses include Campbell and Dietrich (1983), Peters, Pinkus and Askin (1984), Vandell and Thibodeau (1985), Lea and Zorn(1986), Sa-Aadu (1988), and Cunningham and Capone (1990).

indicate that prepayment rates gradually increase with the passing of time. However, no seasoned level is observed. The impact of the Loan-To-Value ratios and housing price mutations is also considered but their impact on the observed prepayment rates is less clear.

Interest rate driven refinance incentives have the most explanatory power, *e.g.* roughly one third of the variability in prepayment activity can be explained by refinance incentive ratios. This explanatory power can be increased to more than 50 percent by including the aforementioned determinants: seasonality, age, loan-to-value ratios and housing prices.

Even though most of the relations observed in this chapter conform to accepted economic theory, not all of them are statistically significant. This is mainly a consequence of the small sample plus the fact that mortgage prepayment is often discouraged by the Dutch tax authorities. More accurate prepayment data is required for an in-depth cross-sectional analysis of prepayment behavior in the Netherlands.

Chapter 10

Summary and concluding remarks

The variety of mortgage finance vehicles has grown rapidly over the last decade. It is now common for mortgages to have multiple embedded options. Without these options a mortgage would be an ordinary non-callable fixed-income security. The most important embedded option is the option to prepay the mortgage. This prepayment option is an American-style call option that gives the borrower (the *mortgagor*) the right to prepay the remaining principal before the contract matures. Similarly, the default possibility of a mortgage can be viewed as a put option. This option gives the borrower the right to sell the house to the lender (the *mortgagee*) at a price equal to the remaining book value of the mortgage.

Due to the resemblance of mortgages to a risk-free asset with various embedded options, recent mortgage valuation models are based on bond option pricing techniques. The starting point in those methodologies is a description of the term structure of interest rates which refers to the relationship between interest rates on fixed-income securities of different maturities. Just like each interest rate derivative, a mortgage contract is an asset whose value depends on the level of interest rates. As with any bond, when interest rates increase, the value of a mortgage decreases. But due to the borrowers' option to prepay, the rise in value is limited when interest rates decrease.

In the Netherlands, mortgage contracts are commonly only partially callable. Within a calendar year, only 10 to 20 percent of the initial loan can be prepaid without cost. Additional prepayments are settled at costs equal to the present value of the difference between future payments of a new mortgage and the existing contract. Sometimes a fixed penalty of 250 to 500 guilders is also charged. These typical Dutch characteristics are described in Chapter 2, alongside various aspects of the Dutch mortgage market. This market has become highly dynamic as reflected by the sharp increase in the variety of loan types, the increasing interest in the secondary market and the substantial growth in the market.

For example, by the end of 1996, 379 billion guilders were outstanding on Dutch residential mortgages, and the number of newly issued residential mortgage contracts increased in 1996 by 42% to 550 thousand (CBS, 1997). More than one third of the new contracts renes, causing the increase in the total amount of mortgage debt to trail the nding. The importance of mortgage replacements and hence prepayments e mortgage pricing algorithms developed in this thesis.

ıg to the newly developed valuation models, Chapter 3 reviews the litera- :c and touches on several relevant mortgage pricing issues. The first such ırtance of the stochastic environment of the mortgage, most commonly de- ynamics of the short-term interest rate. The other main elements are the f interest rates, a model to relate the mortgage rate to this term structure, nent behavior model.

ɔart of Chapter 3 summarizes the economic literature on empirical prepay- whereby the assumption that borrowers try to minimize the market value : is relaxed. Rather than imposing optimal call decisions, these studies ed prepayment behavior. Four main determinants are observed to affect rstly, mortgagors tend to replace the existing mortgage when the prevailing s far enough under the contract rate. This *refinance incentive* is found to ɔortant factor determining prepayment. Secondly, *seasonality* measures the ı the month of the year and prepayment activity, which appears to usually ırch and peak in the autumn months. The remaining two factors, *seasoning* e both related to the age of the contract. *Seasoning* refers to the observed prepayment rates at the beginning of the contract, and *burnout* describes r prepayment rates to decline as the mortgage ages.

re summarized in Chapter 3 reveals that numerical procedures are required ex mortgage contracts. Chapter 4 introduces these numerical methods. ɔrt description of finite difference and simulation methods, this chapter :us to the interest rate tree approach. The principles of mortgage pricing th the help of small-scale examples. The techniques studied in Chapter 4 Chapter 5 to value a more realistic mortgage contract.

ocuses on the impact alternative prepayment rules have on the value and k of mortgages. For this we distinguish between an optimal prepayment rule ss boundary. Under optimality, prepayment is triggered when the present ɔrtgage, if left uncalled, exceeds the outstanding debt plus any refinancing l with refinancing the loan. The moneyness boundary prescribes prepayment s the future costs for the borrower. This latter prepayment rule takes the natives open to the homeowner in consideration, while the optimal rule on the discount rate, *i.e.*, the prevailing mortgage rate does not play an in the prepayment decision! As a consequence, the optimal prepayment be used if the mortgage rate is endogenously derived as a function of the erest rate. The moneyness boundary, on the other hand, can also be applied gage rate is exogenously specified. For example, in Chapter 5, we use both American historical relation between the short-term interest rate and the to value the mortgage contract.

It is common in the Netherlands for mortgages to be only partially callable. The prepayment risk premium included in these contracts reflects the strict prepayment conditions. Consequently, utilizing the empirical relation between Dutch mortgage and short-term interest rates to value freely prepayable mortgages produces values which are under par. Mortgages have fewer prepayment restrictions in the US, such that the prepayment risk premium observed in the US data might be more appropriate here. The mortgage values that are obtained when the American risk premium is used are not only higher than when the Dutch premium is applied, they are also all above par. However, the resulting interest rate sensitivity measures suggest that the value of the mortgage contract decreases with decreasing interest rates. This counter-intuitive result emphasizes the need for an endogenous model to relate the mortgage rate to the short-term interest rate.

In Chapter 5, the endogenous mortgage rate is derived by fixing the mortgage value at origination to the principal value of the loan. This is done for both the optimal prepayment rule and the suboptimal rule which states that the mortgage contract should be replaced if this reduces the future costs for the borrower. This distinction substantially influences the functional relation between the mortgage rate and the short-term interest rate. And also the resulting interest rate risk measures are found to be sensitive to the chosen prepayment rule. Both prepayment rules emphasize, however, the substantial prepayment risk faced by mortgagees when the annual prepayment restrictions are omitted.

To derive the relation between the mortgage rate and the short-term interest rate, Chapter 5 assumes that all interest rates are driven solely by the short-term interest rate. The way the short-term interest rate dynamics are modelled may therefore influence the valuation results. To study this, we specify three empirical one-factor models. Chapter 5 compares the mortgage valuation results from the CIR model with those of a nonlinear model and a nonparametric model, all based on Dutch short-term interest rate data. The different mean-reverting and volatility characteristics explain most of the differences among the three interest rate processes. The interest rate risk measures are found to be especially sensitive to the underlying interest rate process.

Various authors have questioned the single factor assumption and argue that a multi-factor interest rate process is required to model the mortgage rate dynamics. In Chapter 6 we approach this question by studying the empirical relations between short-term interest rates, long-term interest rates and mortgage rates in the Netherlands between 1972 and 1995. Vector AutoRegressive (VAR) techniques are used to analyze the dynamic interactions between these variables. The results are presented for both a stationary specification of the VAR model consistent with mean-reverting interest rate models, and a unit root specification modelling a random walk.

Independent of the VAR specification, Chapter 6 reveals the shortcomings of a one-factor interest rate model. The results indicate that a multi-factor model containing the short-term and long-term interest rate as well as the mortgage rate describes the mortgage rate dynamics more accurately. The VAR parameters estimated in Chapter 6 are utilized to simulate short-term, long-term and mortgage interest rates in a multi-factor mortgage pricing model developed in Chapter 7.

By using simulation techniques, we can integrate complicated stochastic interest rate environments and detailed prepayment restrictions in one mortgage pricing algorithm. Chapter 7 builds on the freely callable contracts studied in Chapter 5 by further considering contracts which are only partially callable within a calendar year, and studying the minimum interest rate guarantee included in many Dutch quotation offers. This latter guarantee reduces the prepayment likelihood and is found to increase the duration of the mortgage contract and decrease the value of the prepayment option. Despite the annual limitations, Chapter 7 shows that these prepayment options are valuable, *e.g.* the value of a 10% penalty-free prepayment option equals one quarter of the value of a prepayment option without any limitations. The value of the 20% penalty-free prepayment option is even equal to half the value of the 100% penalty-free prepayment option!

In order to compare the single factor with the multi-factor approach it was first necessary to bring the VAR analysis period in line with the one-factor model period. Rather than using interest rates observed between January 1972 and December 1995, we switched to rates observed over the 1981-1994 period. This adjustment in the considered period in the multi-factor approach turns out to have a major impact on the interest rate sensitivity of a mortgage contract.

The use of a multi-factor pricing algorithm versus a single factor approach is similarly important. Even though both approaches yield very similar mortgage values, the risk measures differ substantially. Consequently we conclude that, despite the limited effect the alternate interest rate processes have on the mortgage value, the choice for one or the other specific interest rate model has a major impact on the required hedging strategy.

Chapter 8 switches away from mortgages and focus on to the housing market. A repeat sales index is constructed to study the fluctuations in housing prices in the Netherlands between May 1973 and December 1995. This price index for residential property shows a pattern similar to the time series for newly registered mortgages and the growth of total mortgage debt in the Netherlands. Until 1978, both the price of residential property and the total amount of outstanding mortgage debt increased rapidly. After 1978 the real estate bubble exploded. Housing prices dropped in real terms by more than 40 percent over a period of four years. The number and guilder value of newly issued mortgages decreased by a comparable percentage. In the mid-eighties, the number of newly issued mortgage contracts started to increase again, as did property prices. However, the weighted repeat sales index constructed in Chapter 8 shows that, in real terms, the value of the index on December 1995 is still below its all-time maximum of May 1978.

Chapters 5 and 7 assume that the borrower tries to minimize the market value of the loan. The resulting prepayment behavior depends on the mortgage rate at which mortgagors may refinance their loan. American prepayment data reveals that residential mortgage borrowers regularly prepay their mortgage even though the refinancing rate exceeds the contract rate on the existing mortgage. Others fail to exercise their prepayment option when it would be financially wise to do so. The models developed in Chapters 5 and

7 are unable to explain this behavior. Chapter 9 takes an empirical approach and describes the prepayment behavior observed in the Netherlands. The observed prepayment behavior resembles the theoretical results found in Chapters 5 and 7.

The second part of Chapter 9 relates the observed prepayment behavior to various variables suggested by the economic literature outlined in Chapter 3. The seasonal pattern observed in the Dutch data shows that prepayment activity is at it lowest during early spring, after which it increases up to a maximum in July. Due to tax considerations and the approaching expiration of unused annual prepayment options, we observe another peak in December. The refinance incentive is also found to be an important factor in determining prepayment behavior. The age of the contract, the loan-to-value ratio and housing price fluctuations have a less pronounced impact on prepayment rates.

Even though most of the empirical relations presented in Chapter 9 conform to economic theory, not all of them are statistically significant. A more in-depth analysis of Dutch prepayment behavior requires more accurate data, not only regarding the prepayment rates themselves but also for the many variables that influence them.

Nederlandse samenvatting / Dutch summary

Een hypotheek is een lening met onroerend goed als onderpand. Dit onderpand dient als zekerheid voor de geldgever. Mocht de hypotheekgever namelijk in gebreke blijven en niet aan zijn financiële verplichtingen voldoen, dan is de hypotheeknemer gerechtigd het onderpand te verkopen en met de opbrengsten de vorderingen te verrekenen. Merk hierbij op dat de hypotheek*gever* degene is die het onroerend goed in onderpand geeft, terwijl de hypotheek*nemer* de lening verstrekt.

Onder invloed van de lage rente is de Nederlandse hypotheekmarkt de laatste jaren sterk gegroeid. Zo nam het aantal uitstaande hypotheekcontracten in 1996 met 11,16% toe tot 4,34 miljoen. Te zamen vertegenwoordigden deze contracten een hypothecaire schuld van 379 miljard gulden. Met 15,6 miljoen inwoners betekent dit dat per hoofd van de Nederlandse bevolking 24 duizend gulden aan hypotheekschuld uitstond.

Een derde van de 550 duizend contracten die in 1996 werden afgesloten, werden gebruikt om reeds bestaande hypothecaire leningen te vervangen (CBS, 1997). Dit grote aantal oversluitingen benadrukt dat hypotheekgevers veelvuldig gebruik maken van de mogelijkheid om de bestaande hypotheek vroegtijdig af te lossen en te vervangen door een nieuwe lening. Deze opzegmogelijkheid herbergt een risico voor de geldgever. Deze weet namelijk niet of de hypotheekgever de hoofdsom volgens vooraf opgesteld schema zal terugbetalen of dat vervroegd zal worden afgelost.

Zoals voor de meeste vastrentende leningen, geldt ook voor een hypothecaire lening dat de marktwaarde afneemt als de rente stijgt. Bij een rentedaling wordt de waardestijging echter beperkt door de optie om vervroegd af te lossen. In dit geval zullen hypotheekgevers namelijk besluiten om de hypotheek over te sluiten naar een lagere rente. Hypotheeknemers proberen dit negatieve effect op te vangen door aflossingen slechts beperkt boetevrije toe te staan. Zo mag in Nederland per kalenderjaar gewoonlijk slechts 10 tot 20 procent van de orginele hoofdsom boetevrij worden afgelost. Extra aflossingen brengen kosten met zich mee die meestal gelijk zijn aan de contante waarde van het renteverschil. Deze characteristieken die kenmerkend zijn voor Nederlandse hypotheekcontracten worden in hoofdstuk 2 toegelicht.

Naast de standaard hypotheekvormen die in hoofdstuk 2 besproken worden, verschijnen er steeds vaker nieuwe produkten op de markt die de hypotheekgever meer vrijheden bieden. Deze vrijheden bemoeilijken het waarderen van een hypotheek en vragen om een goed waarderingsmodel. En ook het sterk toegenomen aantal oversluitingen, de groeiende hypotheekmarkt en de toegenomen interesse voor de openbare tweedehands markt benadrukken het belang van een accuraat hypotheekwaarderingsmodel.

Een waarderingsmodel voor hypotheken bestaat uit verschillende componenten, waarvan het model dat de rente dynamiek beschrijft een van de meest belangrijke is. Twee andere componenten relateren deze dynamiek aan de rente termijnstructuur en aan de hypotheekrente. Daarnaast is een model dat het aflossingsgedrag beschrijft noodzakelijk.[1]

Voordat wordt overgegaan tot het daadwerkelijk ontwikkelen van een waarderingsmodel, vat hoofdstuk 3 de literatuur op dit gebied samen. In dit hoofdstuk wordt uitgebreid stilgestaan bij de verschillende mogelijkheden om het aflossingsgedrag van de hypotheekgever te modelleren. Hierbij wordt een onderscheidt gemaakt tussen exogene en endogene modellen.

Exogene modellen relateren het waargenomen aflossingsgedrag aan factoren die kenmerkend zijn voor het hypotheekcontract, het onderpand en de hypotheekgever. Natuurlijk worden ook seizoensinvloeden en macro-economische grootheden, zoals rentestanden, meegenomen in deze analyse. Exogene modellen worden op Wall Street veelvuldig toegepast om hypotheekportefeuilles te waarderen. Voor Nederland is deze methode veel minder geschikt omdat het empirisch modelleren van het aflossingsgedrag een uitgebreide dataset vereist die in Nederland niet voorhanden is. In Amerika, waar de markt voor Mortgage-Backed Securities al decennia lang bestaat, zijn deze gegevens in voldoende mate aanwezig om een adequaat aflossingsmodel te construeren. De Amerikaanse bevindingen kunnen echter niet zonder meer worden toegepast in Nederland, daarvoor zijn de verschillen tussen de hypotheekmarkten en contractvormen te groot.

Om Nederlandse hypotheekcontracten te waarderen dienen we gebruik te maken van theoretische modellen waarin het aflossingsgedrag endogeen bepaald wordt. Het aflossingsgedrag is hierbij niet gebaseerd op waarnemingen, maar op beslisregels die worden opgesteld door de analyst. Deze methode is bij uitstek geschikt om het rente-risico voor de geldgever in kaart te brengen en wordt in de hoofdstukken 5 en 7 dan ook gebruikt om de waarde en rentegevoeligheid van een hypotheek te bepalen. De beschouwde hypotheek kan iedere maand volledig worden afgelost. Omdat de waarde van een contract met deze optie niet analytisch bepaald kan worden moeten we gebruik maken van numerieke methoden zoals die in hoofdstuk 4 besproken worden. In dit hoofdstuk staat de zogenaamde renteboombenadering centraal. Aan de hand van eenvoudige voorbeelden worden de sterke en zwakke punten van deze methode toegelicht. De voorbeelden veronderstellen dat een hypotheek wordt afgelost zodra de netto contante waarde van de toekomstige kasstromen hoger is dan de uitstaande schuld. Het aflossingsgedrag wordt in deze optiek dus bepaald door de dynamiek van de rente die gebruikt wordt om de kasstromen te verdisconteren. De nieuwe contractrente waartegen het individu kan herfinancieren speelt hierbij geen enkele rol. In

[1] In principe dient ook een component te worden opgenomen die de mogelijkheid dat de hypotheekgever in gebreke blijft modelleert. Echter zoals in hoofdstuk 3 besproken wordt, is het risico van wanbetaling van ondergeschikt belang. In deze dissertatie wordt dit risico dan ook buiten beschouwing gelaten.

hoofdstuk 5 vergelijken we deze aflosregel met een regel die volledig gebaseerd is op de rente waartegen de hypotheekgever een nieuwe lening kan aangaan.

De aflosregel die gebaseerd is op de disconteringsrente kan alleen gebruikt worden indien de relatie tussen de hypotheekrente en de korte rente endogeen bepaald is. Dat wil zeggen dat de hypotheekrente dusdanig aan de korte rente gerelateerd is dat de netto contante waarde van de lening gelijk is aan de hoofdsom. Als een exogene relatie gebruikt wordt kan het gebeuren dat deze aflosregel de hypotheekgever voorschrijft om zijn contract over te sluiten op een moment dat de hypotheekrente hoger is dan de rente op zijn huidige lening. Dit is niet mogelijk indien de beslissing om al dan niet af te lossen gebaseerd is op de rente waartegen het individu kan herfinancieren. In deze situatie kunnen dan ook exogene specificaties gebruikt worden om de hypotheekrente aan de korte rente te relateren.[2] In hoofdstuk 5 gebruiken we bijvoorbeeld de empirische relaties tussen de korte rente en de hypotheekrente in Nederland en in de Verenigde Staten.

De Nederlandse relatie heeft als voordeel dat de onderliggende rentedynamiek de Nederlandse situatie beschrijft. Een nadeel is echter dat de hypotheekrente gebaseerd is op contracten die in een kalenderjaar slechts gedeeltelijk boetevrij aflosbaar zijn, terwijl het contract dat in hoofdstuk 5 onderzocht wordt iedere maand in z'n geheel mag worden afgelost. Het gebruik van deze historische relatie leidt in sommige gevallen dan ook tot prijzen beneden par. Ondanks het feit dat Amerikaanse contracten minder aflosrestricties kennen blijkt ook de Amerikaanse relatie niet geschikt om het nieuwe hypotheekcontract te analyseren. De waarde van het contract is ditmaal weliswaar in iedere situatie boven par, maar de rente-risico maatstaven worden negatief.

Bovenstaande leert ons dat exogene specificaties weliswaar gebruikt kunnen worden om snel een eerste indruk te krijgen van de waarde en het rente-risico van een hypotheekcontract, maar dat voor een meer nauwkeurige analyse een endogene relatie tussen de korte rente en de hypotheekrente de voorkeur verdient. In hoofdstuk 5 wordt deze endogene relatie afgeleid voor beide aflosregels. De resulterende relaties blijken in sterke mate af te hangen van de onderliggende aflosregel. Zo leidt de aflosregel die gebaseerd is op de disconteringsrente altijd tot een hogere hypotheekrente dan wanneer de aflossingsbeslissing gebaseerd wordt op de rente waartegen de hypotheekgever zijn lening kan herfinancieren. Als deze laatste aflosregel gebruikt wordt zien we dat de endogene relatie tussen beide rentes een vlakker verloop kent.

Om endogeen de hypotheekrente af te leiden hebben we in hoofdstuk 5 verondersteld dat de hypotheekrente slechts afhangt van de korte rente. De manier waarop de dynamiek in de korte rente gemodelleerd is zou daarom de resultaten kunnen beïnvloeden. Om dit te onderzoeken vergelijken we de resultaten van drie verschillende één-factor modellen. Naast het Cox, Ingersoll en Ross model gebruiken we een niet-linear en een niet-parametrisch model. Met name de rente-risico maatstaven blijken gevoelig te zijn voor de manier waarop de dynamiek in de korte rente beschreven wordt.

[2] Bij een exogene specificatie worden historische waarnemingen gebruikt om de functionele relatie tussen de hypotheekrente en de korte rente te bepalen.

De aanname dat de dynamiek in de hypotheekrente beschreven kan worden met een één-factor model is vaak onderwerp van discussie. In hoofdstuk 6 dragen we bij aan deze discussie door de empirische relatie tussen de korte termijn rente, de lange termijn rente en de hypotheekrente in Nederland te onderzoeken. Om de dynamische interacties tussen deze variabelen te bestuderen maken we gebruik van Vector AutoRegressieve (VAR) technieken en presenteren we de resultaten voor zowel een stationaire als een unit root specificatie van het VAR-model. De eerste specificatie bevat drift termen die er voor zorgen dat de verschillende rentes steeds weer terug keren naar hun lange termijn gemiddelde. Dit in tegenstelling tot een unit root proces wat een random walk modelleert.

Beide VAR-specificaties onthullen de tekortkomingen van een één-factor model. Een multi-factor model dat naast de korte termijn rente, ook de lange termijn rente en de hypotheekrente bevat blijkt beter in staat te zijn om de dynamiek in de hypotheekrente te beschrijven. In overeenstemming hiermee wordt in hoofdstuk 7 een multi-factor hypotheekwaarderingsmodel ontwikkeld, waarbij de VAR-parameters geschat in hoofdstuk 6 gebruikt worden om de korte rente, de lange rente en de hypotheekrente te simuleren.

Door simulatie-technieken te gebruiken kunnen we geavanceerde rente dynamieken en gedetailleerde aflosrestricties in een model samenbrengen. Zo bestuderen we in hoofdstuk 7 niet alleen contracten die iedere maand volledig aflosbaar zijn, maar houden we ook rekening met de in Nederland veel voorkomende restrictie dat slechts 10 tot 20 procent van de initiële hoofdsom per kalenderjaar boetevrij mag worden afgelost. Ook bestudeert hoofdstuk 7 de invloed van de rentedalgarantie die garandeert dat het contract wordt afgesloten tegen de laagste rente die voorkomt gedurende de periode dat de hypotheekofferte geldig is. Deze garantie verkleint de kans op vervroegde aflossingen en we vinden dan ook dat de gemiddelde duratie van een hypotheekcontract toeneemt als deze garantie verstrekt wordt.

De VAR-parameters die in het eerste gedeelte van hoofdstuk 7 worden gebruikt om rentes te simuleren, zijn gebaseerd op waarnemingen tussen januari 1972 en december 1995. In hoofdstuk 5 daarentegen, worden observaties tussen januari 1981 en december 1994 gebruikt om de één-factor modellen te schatten. Om beide aanpakken te kunnen vergelijken moeten we dan ook eerst de VAR-parameters aanpassen voor deze kortere subperiode. Deze aanpassing blijkt een groot effect te hebben op de gemeten rentegevoeligheid van een hypotheekcontract. Deze gevoeligheid van de rente-risico maatstaven vinden we ook terug als we de verschillende één-factor modellen vergelijken met de multi-factor aanpak. Dit betekent dat de keuze voor een bepaald rentemodel verstrekkende gevolgen heeft voor de te volgen hedging-strategie.

In hoofdstuk 8 wordt een zijstap gemaakt en richten we de aandacht op de Nederlandse huizenmarkt. Zoals reeds in hoofdstuk 2 naar voren is gebracht, wordt de overgrote meerderheid van alle hypothecaire leningen aangewend om de aanschaf van een eigen woning te financieren. Om te onderzoeken of de ontwikkelingen op de hypotheekmarkt en de woningmarkt elkaar volgen wordt in hoofdstuk 8 een index geconstrueerd die de prijsfluctuaties van Nederlandse koopwoningen tussen mei 1973 en december 1995 weergeeft. Deze index is gebaseerd op prijsontwikkelingen van afzonderlijke huizen. In tegenstelling tot indices die gebaseerd zijn op gemiddelde verkoopprijzen of op de mediaan van deze prijzen, is de in hoofdstuk 8 beschreven index nauwelijks gevoelig voor de samenstelling van huizen die in een bepaalde periode verkocht worden.

De huisprijzenindex toont een beeld dat overeenstemt met de fluctuaties op de hypotheekmarkt. Tot 1978 stegen bijvoorbeeld zowel de huisprijzen als de totaal uitstaande hypothecaire schuld. En ook de in eenstorting van de huizenmarkt eind jaren zeventig, begin jaren tachtig is weerspiegeld in de fluctuaties op de hypotheekmarkt. In een tijdsbestek van minder dan vier jaar (1978-1982) daalde de huisprijzen in reële termen met meer dan 40%. Eind 1995 is de huizenmarkt deze klap nog steeds niet te boven en ligt het reële prijsniveau van woningen nog onder dat van 1978.

De prijsfluctuaties van koopwoningen worden in hoofdstuk 9 gebruikt om empirisch het aflossingsgedrag van Nederlandse hypotheekgevers te onderzoeken. In tegenstelling tot de hoofdstukken 5 en 7 veronderstelt dit hoofdstuk niet dat een hypotheekgever de marktwaarde van zijn of haar hypotheekcontract probeert te minimaliseren. In plaats daarvan richt hoofdstuk 9 zich op het waargenomen aflossingsgedrag van een kleine groep Nederlandse hypotheekgevers. Dit gedrag, dat grote overeenkomsten blijkt te vertonen met de resultaten uit de theoretische hoofdstukken 5 en 7, wordt gerelateerd aan verschillende factoren die in hoofdstuk 3 beschreven zijn.

In de Nederlandse aflossingsdata valt een seizoenspatroon te herkennen dat in grote lijnen overeenstemt met de Amerikaanse resultaten. Ook in ons land wordt in het voorjaar minder afgelost dan in de rest van het jaar. Het maximum wordt in juli bereikt en niet, zoals in de Verenigde Staten, in de herfstmaanden. Het grootste verschil tussen het Nederlandse en Amerikaanse seizoenspatroon vinden we echter in december. Daar waar in de Verenigde Staten de aflossingen afnemen, zien we dat in Nederland in december meer dan gemiddeld wordt afgelost. De reden hiervoor is gelegen in het feit dat de aflosrestricties in Nederland normaliter gebonden zijn aan een kalenderjaar. De meeste hypothekencontracten laten bijvoorbeeld niet toe dat er meer dan 10 procent van de initiële hoofdsom per kalenderjaar boetevrij wordt afgelost. Mocht deze 10 procent's norm in december nog niet bereikt zijn, dan zullen veel mensen besluiten om alsnog van deze optie gebruik te maken.

Naast het seizoenspatroon vinden we in hoofdstuk 3 dat de rente een belangrijke factor is om het waargenomen aflossingsgedrag te verklaren. Er blijkt duidelijk een prikkel te zijn om te herfinancieren als de rente laag is. De leeftijd van het contract, de relatie tussen de omvang van de lening en de waarde van het onderliggende pand en ook fluctuaties in huisprijzen hebben een veel minder duidelijke invloed op het aflossingsgedrag. Het aantal waarnemingen is echter te beperkt om hieruit te concluderen dat deze variabelen het aflossingsgedrag niet beïnvloeden. Voor een grondige empirische studie zijn meer gegevens vereist, niet alleen met betrekking tot het aflossingsgedrag, maar ook omtrent de variabelen die dit gedrag beïnvloeden.

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Curriculum Vitae

Arjan van Bussel was born on September 21^{st}, 1970 in Asten, the Netherlands. He graduated with distinction from Maastricht University in 1993. His Masters thesis, "*The duration of index-linked bonds*", was researched and written during an internship at the public pension fund ABP in Heerlen, the Netherlands. This thesis won both the Maastricht University student-research award in Economics and the AXA-Equity & Law prize for the best Masters thesis in Finance. After completing a one-year Japan Studies Program at the International Christian University in Tokyo, Arjan van Bussel returned to Maastricht University to work for the Finance Department where he wrote this Ph.D. dissertation on mortgage pricing. Since January 1^{st}, 1998, he has been working at the B.V. NIB Mortgage-Backed Assets, a 100% subsidiary of *De Nationale InvesteringsBank* (DNIB).

Arjan van Bussel is op 21 september 1970 in Asten geboren. In 1993 studeerde hij cum laude af aan de Faculteit der Economische Wetenschappen en Bedrijfskunde van de Universiteit Maastricht, toen nog Rijksuniversiteit Limburg genaamd. Zijn afstudeerscriptie, "*De duratie van een indexlening*", is gebaseerd op een stage bij het Algemeen Burgerlijk Pensioenfonds (ABP). Met deze scriptie won hij zowel de studentenprijs van de economische faculteit van de Universiteit Maastricht als de AXA Equity & Law scriptieprijs. Van september 1992 tot en met augustus 1993 studeerde Arjan van Bussel aan de International Christian University in Tokyo, alwaar hij het Japan Studies Program met goed resultaat afrondde. In september 1993 trad hij als assistent in opleiding in dienst bij de sectie financiering van de Universiteit Maastricht. Hij werkte daar aan verschillende onderzoeksprojecten, waarvan dit proefschrift het resultaat is. Vanaf 1 januari 1998 is hij werkzaam bij B.V. NIB Mortgage-Backed Assets, een 100% dochter van *De Nationale InvesteringsBank* (DNIB).